

Deriving principle channel metrics from bank and long-profile geometry with the R-package cmgo

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Abstract

Landscape patterns result from landscape forming processes. This link can be exploited in geomorphological research by reversely analyzing the geometrical content of landscapes to develop or confirm theories of the underlying processes. Since rivers represent a dominant control on landscape formation, there is a particular interest in examining channel metrics in a quantitative and objective manner. For example, river cross-section geometry is required to model local flow hydraulics which in turn determine erosion and thus channel dynamics. Similarly, channel geometry is crucial for engineering purposes, water resource management and ecological restoration efforts. These applications require a framework to capture and derive the data. In this paper we present an open-source software tool that performs the calculation of several channel metrics (length, slope, width, bank retreat, knickpoints, etc.) in an objective and reproducible way based on principle bank geometry that can be measured in the field or in a GIS. Furthermore, the software provides a framework to integrate spatial features, for example the abundance of species or the occurrence of knickpoints. The program is available <https://github.com/AntoniusGolly/cmgo> and is free to use, modify and redistribute under the terms of the GNU General Public License version 3 as published by the Free Software Foundation.

23 1. Introduction

24 Principle channel metrics, for example channel width or gradient, convey immanent information
25 that can be exploited for geomorphological research (Wobus et al. 2006; Cook et al. 2014) or
26 engineering purposes (Pizzuto 2008). For example, a snap-shot of the current local channel
27 geometry can provide an integrated picture of the processes leading to its formation, if interpreted
28 correctly and examined in a statistically sound manner (Ferrer-Boix et al. 2016). Repeated surveys,
29 as time-series of channel gradients, can reveal local erosional characteristics that sharpen our
30 understanding of the underlying processes and facilitate, inspire and motivate further research
31 (Milzow et al. 2006). However, these geometrical measures are not directly available. Typically,
32 the measurable metrics are limited to the position of features, such as the channel bed or water
33 surface, or the water flow path or thalweg in two- or three-dimensional coordinates. The data can
34 be either collected during field surveys with GPS or total stations or through remote sensing, with
35 the need of post-processing for example in a GIS (geographical information system). To effectively
36 generate channel metrics such as channel width, an objective and reproducible processing of the
37 geometric data is required, especially when analyzing the evolution of channel metrics over time.
38 For river scientists and engineers a convenient processing tool should incorporate a scale-free
39 approach applicable to a broad spectrum of environments. It should be easy to access, use and
40 modify, and generate output data that can be integrated in further statistical analysis. Here, we
41 present a new algorithm that meets these requirements and describe its implementation in the R
42 package *cmgo* (<https://github.com/AntoniusGolly/cmgo>). The package derives a reference
43 (centerline) of one or multiple given channel shapes and calculates channel length, local and
44 average channel width, local and average slope, knickpoints based on a scale-free approach
45 (Zimmermann et al. 2008), local and average bank retreat, or the distances from the centerline
46 respectively, as well as allows to project additional spatial metrics to the centerline.

47 2. Literature review

48 Computer-aided products for studying rivers have a long tradition, and solutions for standardized
49 assessments include many disciplines, as for example for assessing the ecological status of rivers
50 (Asterics 2013) or for characterizing heterogeneous reservoirs (Lopez S., Cojan I., Rivoirard J.
51 2008). There are also numerous efforts to derive principle channel metrics from remote or in-situ
52 measurements of topography or directly of features such as channel banks. Available products,
53 which we review in detail next (Table 1), are helpful for many scientific applications and are used

54 by a large community. However, they often do not provide the degree of independency,
55 transparency or functionality that is necessary to fit the versatile requirements of academic or
56 applied research and thus the call for software solutions remains present (Amit 2015). The currently
57 available solutions can be separated into two groups: extensions for GIS applications and
58 extensions for statistical programming languages. The first group incorporates programs that are
59 published as extensions for the proprietary GIS software ArcMap (ESRI 2017), which are generally
60 not open source and are thus lacking accessibility and often transparency and modifiability.
61 Furthermore, the individual solutions lack functionality. For example, the *River Width Calculator*
62 (Mir et al. 2013) calculates the average width of a given river (single value), without providing
63 spatially resolved information. The toolbox *Perpendicular Transects* (Ferreira 2014) is capable of
64 deriving channel transects locally, which are generally suitable for calculating the width. However,
65 the required centerline to which the orthogonals are computed is not generated within the tool itself.
66 Thus, the tool does not represent a full stack solution. Similarly, the *Channel Migration Toolbox*
67 (Legg et al. 2014), *RivEX* (Hornby 2017) and *HEC-GeoRAS* (Ackerman 2011) require prerequisite
68 products – a centerline – to compute transects and calculate the width. A centerline could be created
69 with the toolbox *Polygon to Centerline* (Dilts 2015), but manual post-processing is required to
70 ensure that lines connect properly. Further, the details of the algorithm are poorly documented and
71 intermediate results are not accessible, making it difficult to understand the data quality. Apart from
72 this, all of these products are dependent on commercial software, are bound to a graphical user
73 interface (not scriptable) and cannot be parametrized to a high degree.

74 The second group of solutions represent extensions for statistical scripting languages. The full stack
75 solution *RivWidth* (Pavelsky & Smith 2008) is written as a plugin for IDL, a data language with
76 marginal use (Tiobe 2017), which recently became member-restricted. The program requires two
77 binary raster masks, a channel mask and a river mask, which need to be generated in a pre-
78 processing step, using for example a GIS. Bank geometry obtained from direct measurements, for
79 example from GPS surveys, do not represent adequate input. As a result of the usage of pixel-based
80 data – which in the first place does not properly represent the nature of the geometrical data –
81 computational intensive transformations are necessary, resulting in long computation times (the
82 authors describe up to an hour for their example). More importantly, the centerline position depends
83 on the resolution of the input rasters, and thus is scale-dependent. Good results can only be obtained
84 when the pixel size is at least an order of magnitude smaller than the channel width. The MATLAB
85 toolbox *RivMap* also works with raster data. It is well documented and has a scientific reference
86 (Schwenk et al. 2017). However, intermediate results are not accessible. For example the transects
87 used for generating the local width are not accessible. Thus, the tool lacks an important mechanism

88 to validate its results. However, since *RivMap* represents the best documented and most versatile
89 tool, we choose it to compare our results to in the section 8. *Evaluation of the data quality*.
90 To quantify channel bank retreat for repeated surveys, tools designed for other purposes could
91 potentially be used. Examples are *DSAS* (Thieler et al. 2009) and *AMBUR* (Jackson 2009), designed
92 for analyzing migrating shore lines. These tools also require a baseline that is not derived by the
93 program. *AMBUR*, scripted in the open-source environment R (Jackson 2009) could be adapted to
94 channels. However, we judge its approach to derive transects to be unreliable and unsuitable for
95 rivers, as the transects do not cross the channel orthogonally, leading to implausible results
96 especially in regions with large curvature. A further correction step is included to alleviate this
97 problem, but the resulting distances of the baselines seem arbitrary. Thus, although the tool is
98 among the best documented and accessible solutions currently available, its algorithm is not
99 suitable for generating channel metrics in an objective manner. We conclude that none of the
100 available approaches combines the criteria of being a tool for objectively deriving channel metrics,
101 being easy and free to use and modify and allowing a high degree of parametrization and fine-
102 tuning.

Name of the tool	Platform	Data format	Last updated	Free to use ¹⁾	Free to modify ²⁾	Configurable	Full-stack solution ³⁾	Scientific reference	Note
<i>cmgo (this paper)</i>	R	Vector	July 2017	yes, yes	yes	yes	yes	yes ⁴⁾	
<i>RiverWidthCalculator</i>	ArcMap	Raster	June 2013	no, yes	no	no	no	yes	• single, average value for a stream
<i>Perpendicular Transects</i>	ArcMap	Vector	Dec 2014	no, yes	yes	limited, no smoothing	no	no	• weak output on non-smooth centerlines
<i>Channel Migration Toolbox</i>	ArcMap	Vector	Oct 2014	no, yes	no	limited	no	yes	• fails silently
<i>RivEX</i>	ArcMap	Vector	Feb 2017	no, no	no	yes	no	no	• works only on demo data
<i>HEC-GeoRAS</i>	ArcMap	Raster	July 2017	no, yes	no	yes	no	no	• only verified until ArcMap 10.2
<i>Polygon to Centerline</i>	ArcMap	Vector	Nov 2016	no, yes	no	limited, no smoothing	no	no	• weak output for high-resolution bank geometry
<i>Fluvial Corridor Toolbox</i>	ArcMap	Vector	Jan 2016	no, yes	no	yes	no	yes	• cannot be applied on the raw data, requires pre-vectorization of channel features
<i>Stream Restoration Toolbox</i>	ArcMap	Vector	⁵⁾	no, yes	no	very limited	no	no	• limited functionality • very unstable
<i>RivWidth</i>	IDL	Raster	May 2013	no, yes	yes	⁵⁾	⁵⁾	yes	• limited access due to IDL license
<i>DSAS</i>	ArcMap	Vector	Dec 2012	no, yes	no	yes	no	yes	• primarily designed for coast lines
<i>AMBUR</i>	R	Vector	June 2014	yes, yes	yes	limited	no	yes	• no multi-temporal analyses allowed
<i>RivMap</i>	MATLAB	Raster	Apr 2017	no, yes	yes	yes	limited	yes	• primarily for large scale river systems • fails silent on errors

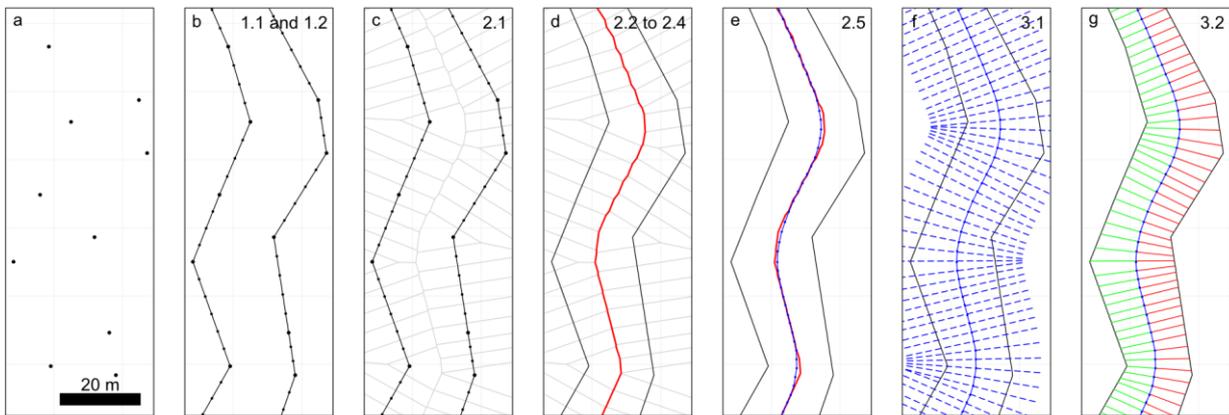
103 **Table 1: overview of existing products, 1) the two values indicate free use of framework (first) and plugin (second value), 2) a**
104 **product is considered free to modify if users can access and edit the source code and a license explicitly allows users to do so, 3)**
105 **a product is considered a full-stack solution if it performs all steps from the bank geometry to the derived channel metrics, 4)**
106 **relies on the publication of this manuscript, 5) gray cells indicate that no information could be gathered by the time of writing**
107 **this.**

108 3. Description of the algorithm

109 Our aim in this paper is to develop a program that does not have the shortcomings of previous
 110 approaches and offers a transparent and objective algorithm. The algorithm (full list of steps in
 111 Table 2 and visualization in Figure 1) has two main parts. First, a centerline of the channel – defined
 112 by the channel bank points – is derived and second, from this centerline the metrics – channel
 113 length, width and gradient (the latter only if elevation is provided) – are calculated. Furthermore,
 114 this reference centerline allows for projecting secondary metrics (as for example the occurrence of
 115 knickpoints) and performing temporal comparisons (more information on temporal analyses in
 116 section 5).

Step	Description	Function
1.1	Generate polygon from bank points	CM.generatePolygon()
1.2	Interpolate polygon points	
2.1	Create Voronoi polygons and convert to paths	CM.calculateCenterline())
2.2	Filter out paths that do not lie within channel polygon entirely	
2.3	Filter out paths that are dead ends (have less than 2 connections)	
2.4	Sorting of the centerline segments to generate centerline	
2.5	Spatially smooth the centerline segments (mean filter)	
2.6	Measure the centerline's length and slope	
2.7	Project elevation to the centerline points (optional)	
3.1	Derive transects of the centerline	CM.processCenterline()
3.2	Calculate intersections of the centerline with the banks	
3.3	Project custom geospatial data onto centerline (optional)	
3.4	Calculate knickpoints based on scale-free approach (Zimmermann et al. 2008)	

117 **Table 2: full list of steps of the algorithm of the package *cmgo* and their functions**



118
 119 **Figure 1: visualization of the work flow of the package, a) the channel bank points represent the data input, b) a polygon is**
 120 **generated where bank points are linearly interpolated, c-d) the centerline is calculated via Voronoi polygons, e) the centerline**
 121 **is spatially smoothed with a mean filter, f) transects are calculated, g) the channel width is derived from the transects.**

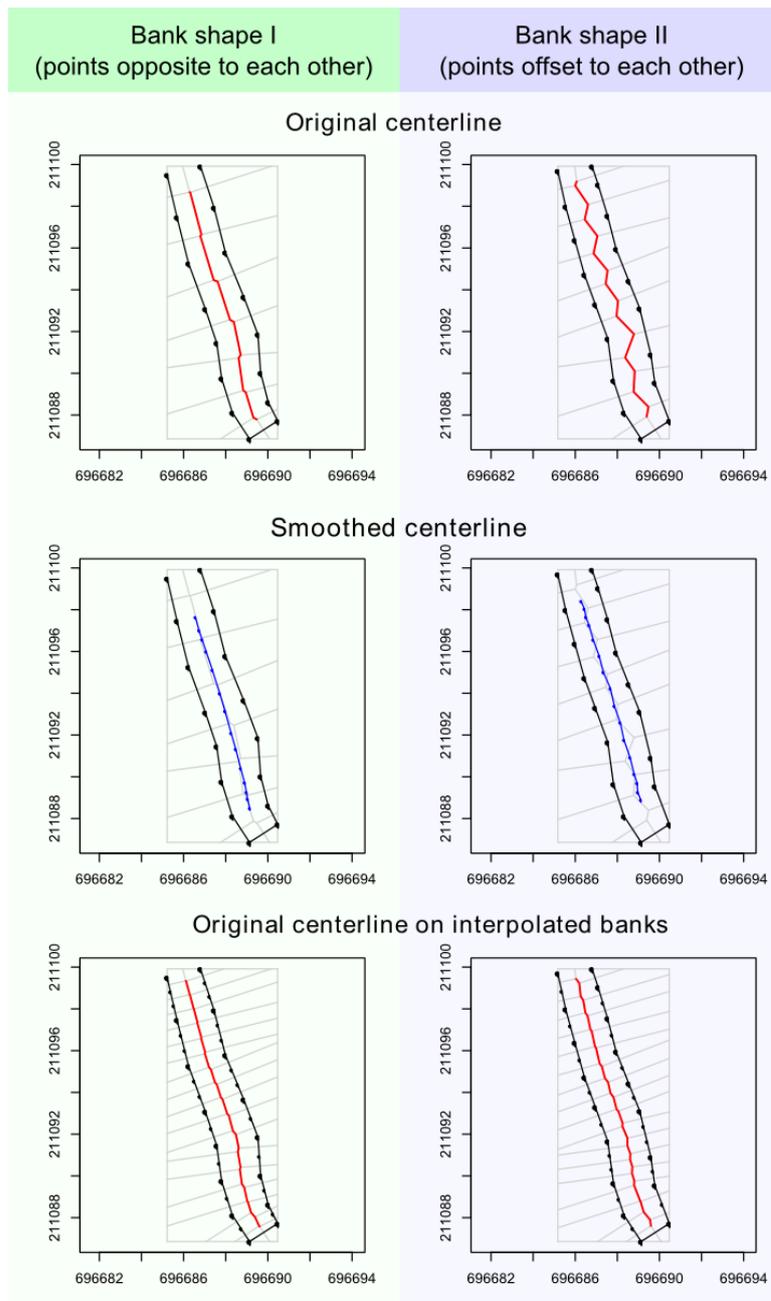
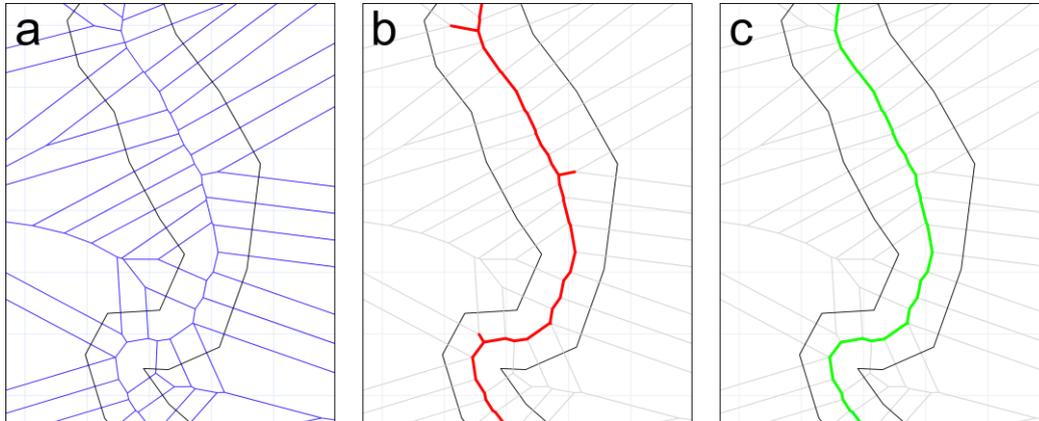


Figure 2: the plot shows two digitizations (Bank shape I and II) of the same channel stretch. They differ only in the arrangement of bank points which are mainly opposite (Bank shape I, left column) or offset (Bank shape II, right column) to each other. One can see how the offset negatively influences the shape of the centerline (top row). The problem can be overcome by smoothing the centerline a-posteriori (middle row) or interpolating between the bank points a-priori (bottom row). A combination of both methods is recommended and set as the default in cmgo.

122 It follows a detailed description of all steps of the algorithm. In step 1.1, the algorithm creates a
 123 polygon feature from the bank points (Figure 1b), where the points are linearly interpolated (step
 124 1.2) to increase their spatial resolution. This is a crucial step for improving the shape of the resulting
 125 centerline – even for straight channel beds (see Fig. 2). From the interpolated points, Voronoi
 126 polygons (also called Dirichlet or Thiessen polygons) are calculated (2.1, Figure 1c). In general,
 127 Voronoi polygons are calculated around center points (here the bank points) and denote the areas
 128 within which all points are closest to that center point. Next, the polygons are disassembled into

129 single line segments. The segments in the center of the channel polygon form the desired centerline
130 (see Figure 1c). The algorithm then filters for these segments by first removing all segments that
131 do not lie entirely within the channel banks (step 2.2, Figure 3b). In a second step, dead ends are
132 removed (step 2.3, Figure 3c). Dead ends are segments that branch from the centerline but are not
133 part of it, which are identified by the number of connections of each segment. All segments, other
134 than the first and the last, must have exactly two connections. The filtering ends successfully if no
135 further dead ends can be found. In step 2.4, the centerline segments are chained to one consistent
136 line, the “original” centerline. In the final step 2.5 of the centerline calculation, the generated line
137 is spatially smoothed (Figure 1e) with a mean filter with definable width (see section 4.2) to correct
138 for sharp edges and to homogenize the resolution of the centerline points. This calculated centerline,
139 the “smoothed” centerline, is the line feature representation of the channel – for example it
140 represents its length, which is calculated in step 2.6. If elevation data is provided with the bank
141 point information (input data) the program also projects the elevation to the centerline points and
142 calculates the slope of the centerline in step 2.7. The program also allows projecting custom
143 geospatial features to the centerline – for example the abundance of species, the occurrence of
144 knickpoints, etc. – if in hand (see section 4.2). Projecting means here that elevation information or
145 other spatial variables are assigned to the closest centerline points.



146
147 **Figure 3: the filtering of the Voronoi segments (a) the final centerline by first taking only segments that lie fully within the**
148 **channel polygon (b) and then filter out dead ends (c).**

149 To calculate the channel metrics based on the centerline, channel transects are derived (step 3.1).
150 Transects are lines perpendicular to a group of centerline points. In step 3.2, the intersections of the
151 transects with the banks are calculated (Figure 1g). When transects cross the banks multiple times,
152 the crossing point closest to the centerline is used. The distance in the x-y-plane between the
153 intersections represent the channel width at this transect. In addition to the width, the distances
154 from the centerline points to banks are stored separately for the left and the right bank.

155 4. Implementation and execution

156 The program is written as a package for the statistical programming language R (Yan et al. 2011).
157 The program can be divided into three main parts which are worked through during a project: 1.
158 initialization (loading data and parameters, section 4.1), 2. data processing (calculating centerline
159 and channel metrics, section 4.2), and 3. review of results (plotting or writing results to file,
160 section 4.3).

161 4.1. INITIALIZATION: INPUT DATA AND PARAMETERS

162 The package *cmgo* requires basic geometrical information of the points that determine a channel
163 shape – the bank points (Figure 1a) – while in addition of the coordinates the side of the channel
164 must be specified for each point. In principle, a text file with the three columns “x”, “y” and “side”
165 represent the minimum input data required to run the program (Codebox 1). The coordinates “x”
166 and “y” can be given in any number format representing Cartesian coordinates, and the column
167 “side” must contain strings (e.g. “left” and “right”) as it represents information to which of the
168 banks the given point is associated. Throughout this paper we refer to left and right of the channel
169 always in regard to these attributes. Thus, the user is generally free to choose which side to name
170 “left”. However, we recommend to stick to the convention to name the banks looking in
171 downstream direction. In addition, a fourth column “z” can be provided to specify the elevation of
172 the points. This allows for example for the calculation of the channel gradient. Note, that the order
173 of the bank points matter. By default it is expected that the provided list are all bank points in
174 upstream direction. If one – this can be the case when exporting the channel bed from a polygon
175 shape – or both banks are reversed, the parameters `bank.reverse.left` and/or `bank.reverse.right`
176 should be set `TRUE`. The units of the provided coordinates can be specified in the parameter
177 `input.units` and defaults to `m` (meters).

```
Name POINT_X POINT_Y  
right 401601.0819 3106437.335  
right 401586.5327 3106406.896  
right 401568.3238 3106383.586  
right 401558.4961 3106364.129  
...  
left 401621.4337 3106431.134  
left 401602.9913 3106405.991  
left 401574.6073 3106352.232  
left 401582.2671 3106323.134  
...
```

178
179 **Codebox 1: example of input data table with columns side and x,y,z-coordinates.**

180 The data can be either collected during field surveys with GPS or total stations or through remote
181 sensing techniques with further digitizing for example in a GIS. In the latter case the data needs to

182 be exported accordingly. The input can be given in any ASCII table format. By default, the program
 183 expects a table with tab-delimited columns and one header line with the column names `POINT_X`,
 184 `POINT_Y` and `POINT_Z` (the coordinates of the bank points) where the z component is optional and `Names`
 185 (for the side). The tab delimiter and the expected column names can be changed in the parameters
 186 (see SM I for details). The input file(s) – for multiple files see also section 5 – have to be placed in
 187 the input directory specified by the parameter `input.dir` (defaults to `./input`) and can have any file
 188 extension (`.txt`, `.csv`, etc.). The data reading function iterates over all files in that directory and
 189 creates a data set for each file.

190 All the data and parameters used during runtime are stored in one variable of type list (see R
 191 documentation): the global data object. Throughout the following examples this variable is named
 192 `cmgo.obj` and its structure is shown in Codebox 2. The global data object also contains the parameter
 193 list, a list of more than 50 parameters specifying the generation and plotting of the model results.
 194 The full list of parameters with explanations can be found in SM I.

```

cmgo.obj = list(
  data = list(
    set1 = list(
      filename           = "input.1.csv", # corresponding filename
      channel            = list(),       # input coordinates of banks
      polygon.bank.interpolate = TRUE,
      polygon            = list(),       # polygon object
      polygon.bank.interpolate.max.dist = 6,
      cl                 = list(),       # centerlines (original and smoothed)
      metrics            = list(),       # calculated metrics (width, etc.)
    ),
    set2 = list()           # survey 2
    # ...
  ),
  par = list()             # all model and plotting parameters
)

```

195

196 **Codebox 2: structure of the global data object containing data and parameters.**

197 To create this object, the function `CM.ini(cmgo.obj, par)` is used. Initially, the function builds a
 198 parameter object based on the second argument `par`. If the `par` argument is left empty, the default
 199 configuration is loaded. Alternatively, a parameter filename can be specified (see the R
 200 documentation of `CM.par()` for further information). Once the parameter object is built, the function
 201 fills the data object by the following rules (if one rule was successful, the routine stops and returns
 202 the global data object):

1. If `cmgo.objparworkspace.read` is `TRUE` (default) the function looks for an `.RData` workspace
 204 file named `cmgo.objparworkspace.filename` (defaults to `./user_workspace.RData`). Note:
 205 there will be no such workspace file once a new project is started, since it needs to be saved
 206 by the user with `CM.writeData()`. If such a workspace file exists the global data object is
 207 created from this source, otherwise the next source is tested.

208 2. If data input files are available in the directory `cmgo.objparinput.dir` (defaults to `./input`)
209 the function iterates over all files in this directory and creates the data object from this
210 source (see section "Input data" above for further information on the data format). In this
211 case the program starts with the bank geometry data set(s) found in the file(s). Otherwise
212 the next source is tested.

213 3. If the `cmgo.obj` argument is a string or `NULL`, the function will check for a demo data set with
214 the same name or "demo" if `NULL`. Available demo data sets are "demo", "demo1", "demo2"
215 and "demo3" (section 7).

216 `CM.ini()` returns the global data object which must be assigned to a variable, as for example
217 `cmgo.obj = CM.ini()`. Once the object is created, the data processing can be started.

218 4.2. CONTROLLING THE DATA PROCESSING

219 The processing includes all steps from the input data (bank points) to the derivation of the channel
220 metrics (Figure 1). Next, we describe the parameters that are relevant during the processing
221 described in section 0. When generating the channel polygon the spatial resolution of the bank
222 points is increased by linear interpolation (Figure 1b) in order to increase the resulting resolution
223 of the channel centerline. The interpolation is controlled through the parameters
224 `cmgo.objparbank.interpolate` and `cmgo.objparbank.interpolate.max.dist`. The first is a Boolean
225 (`TRUE/FALSE`) that enables or disables the interpolation (default `TRUE`). The second determines the
226 maximum distance of the interpolated points. The unit is the same as of the input coordinates, which
227 means, if input coordinates are given in meters, a value of 6 (default) means that the points have a
228 maximum distance of 6 meters to each other. These parameters have to be determined by the user
229 and are crucial for the centerline generation. Guidance of how to select and test these parameters
230 can be found in paragraph 6. *Technical fails and how to prevent them.*

231 During the filtering of the centerline paths, there is a routine that checks for dead ends. This routine
232 is arranged in a loop that stops when there is no further paths to remove. In cases, where the
233 centerline paths exhibit gaps (see section 6) this loop would run infinitely. To prevent this, there is
234 a parameter `bank.filter2.max.it` (defaults to 12) that controls the maximum number of iterations
235 used during the filtering.

236 In the final step of the centerline calculation, the generated line gets spatially smoothed with a mean
237 filter (Figure 1e) where the width of smoothing in numbers of points can be adjusted through the
238 parameter `cmgo.objparcenterline.smoothing.width` (by default equals 7). Note, that the degree of
239 smoothing has an effect on the centerline length (e.g. a higher degree of smoothing shortens the
240 centerline). Similar to the coast line paradox (Mandelbrot 1967), the length of a channel depends

241 on the scale of the observations. Technically, the length diverges to a maximum length at an
242 infinitely high resolution of the bank points. However, practically there is an appropriate choice of
243 a minimum feature size where more detail in the bank geometry only increases the computational
244 costs without adding meaningful information. The user has to determine this scale individually and
245 should be aware of this choice. To check the consequences of this choice, the decrease in length
246 due to smoothing is saved as fraction value in the global data object under
247 `cmgo.obj$data[[set]]$cl$length.factor`. A value of 0.95 means that the length of the smoothed
248 centerline is 95% the length of the original centerline paths. For the further calculations of transects
249 and channel metrics by default the smoothed version of the centerline is used.

250 The program will project automatically the elevation of the bank points to the centerline if elevation
251 information is provided in the input files (z component of bank points, see paragraph 4.1). Also
252 additional custom geospatial features – if available to the user – can be projected to the centerline,
253 for example the abundance of species, the occurrence of knickpoints, etc. Additional features are
254 required to be stored in the global data object as lists with x,y-coordinates (Codebox 3) to be
255 automatically projected to the centerline. Projecting here means that features with x,y-coordinates
256 are assigned to the closest centerline point. The distance and the index of the corresponding
257 centerline point are stored within the global data object.

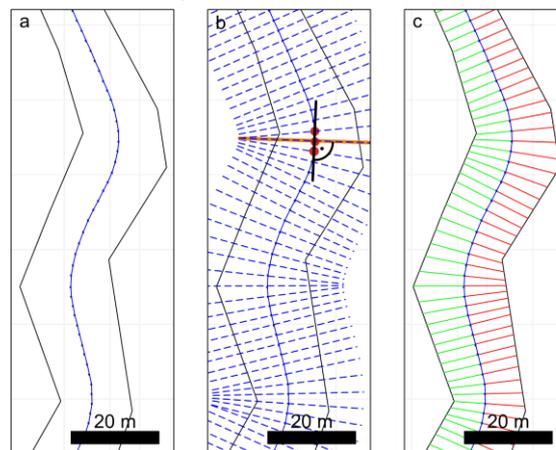
```
cmgo.obj$data[[set]]$features = list(  
  custom_feature_1 = list(  
    x = c(),  
    y = c()  
  ),  
  knickpoints = list(  
    x = c(),  
    y = c()  
  )  
)
```

258

259 **Codebox 3: the format of secondary spatial features to be projected to the centerline.**

260 To calculate the channel metrics based on the centerline channel transects are derived. Transects
261 are lines perpendicular to a group of centerline points, where the size of that group is defined by
262 the parameter `cmgo.objpartransects.span`. By default this span equals three, which means for each
263 group of three centerline points a line is created through the outer points of that group to which the
264 perpendicular – the transect – is calculated (see Figure 4b). The number of resulting transects equals
265 the number of centerline points and for each centerline point the width `w` and further metrics are
266 calculated (see Codebox 4). The distances of the centerline points to the banks is stored separately
267 for the left and the right bank (`d.r.` and `d.l.`), as well as factor (`r.r` and `r.l`) of +/- 1 representing the
268 side of the bank with regard to the centerline. Normally, looking downstream the right bank is also
269 right to the centerline (value of -1) and the left bank is always left to the centerline (value of +1).

270 However, when using a reference centerline to compare different channel surveys, the centerline
 271 can be outside the channel banks for which the metrics are calculated. To resolve the real position
 272 of the banks for tracing their long-term evolution (e.g. bank erosion and aggradation) the factors of
 273 `r.r.` and `r.l` must be considered for further calculations (see also section 5.1). A sample result for a
 274 reach of a natural channel is provided in Figure 5.



275
 276 **Figure 4: from the smoothed centerline (a) transects are calculated (b) by taking a group of centerline points and create a line**
 277 **through the outer points. The perpendicular to that line is the transect. The algorithm now checks for the intersection of the**
 278 **transect with the channel banks (c).**

```

$metrics$tr      # linear equations of the transects
$metrics$cp.r   # coordinates of crossing points transects / right bank
$metrics$cp.l   # coordinates of crossing points transects / left bank
$metrics$d.r    # distance of reference centerline point / right bank
$metrics$d.l    # distance of reference centerline point / left bank
$metrics$w      # channel width
$metrics$r.r    # direction value: -1 for right, +1 for left to the centerline
$metrics$r.l    # direction value: -1 for right, +1 for left to the centerline
$metrics$diff.r # difference between right bank point of actual time series and right bank
                # point of reference series
$metrics$diff.l # difference between left bank point of actual time series and left
                # bank point of reference series
  
```

279
 280 **Codebox 4: the calculated metrics and their variable names (stored in the global data object under `cmgo.obj$data[[set]]`).**

281 4.3. REVIEW RESULTS: PLOTTING AND WRITING OF THE OUTPUTS

282 After the metrics are calculated and stored within the global data object, the results can be plotted
 283 or written to data files. The plotting functions include a map-like type plan view plot
 284 (`CM.plotPlanView()`), a plot of the spatial evolution of the channel width (`CM.plotWidth()`) and a plot
 285 of the spatial and temporal evolution of the bank shift (`CM.plotMetrics()`). All plotting functions
 286 require a data set to be specified that is plotted (by default “set1”). Additionally, all plotting
 287 functions offer ways to specify the plot extent to zoom to a portion of the stream for detailed
 288 analyses. In the plan view plot, multiple ways exists to define the plot region (also called extent),

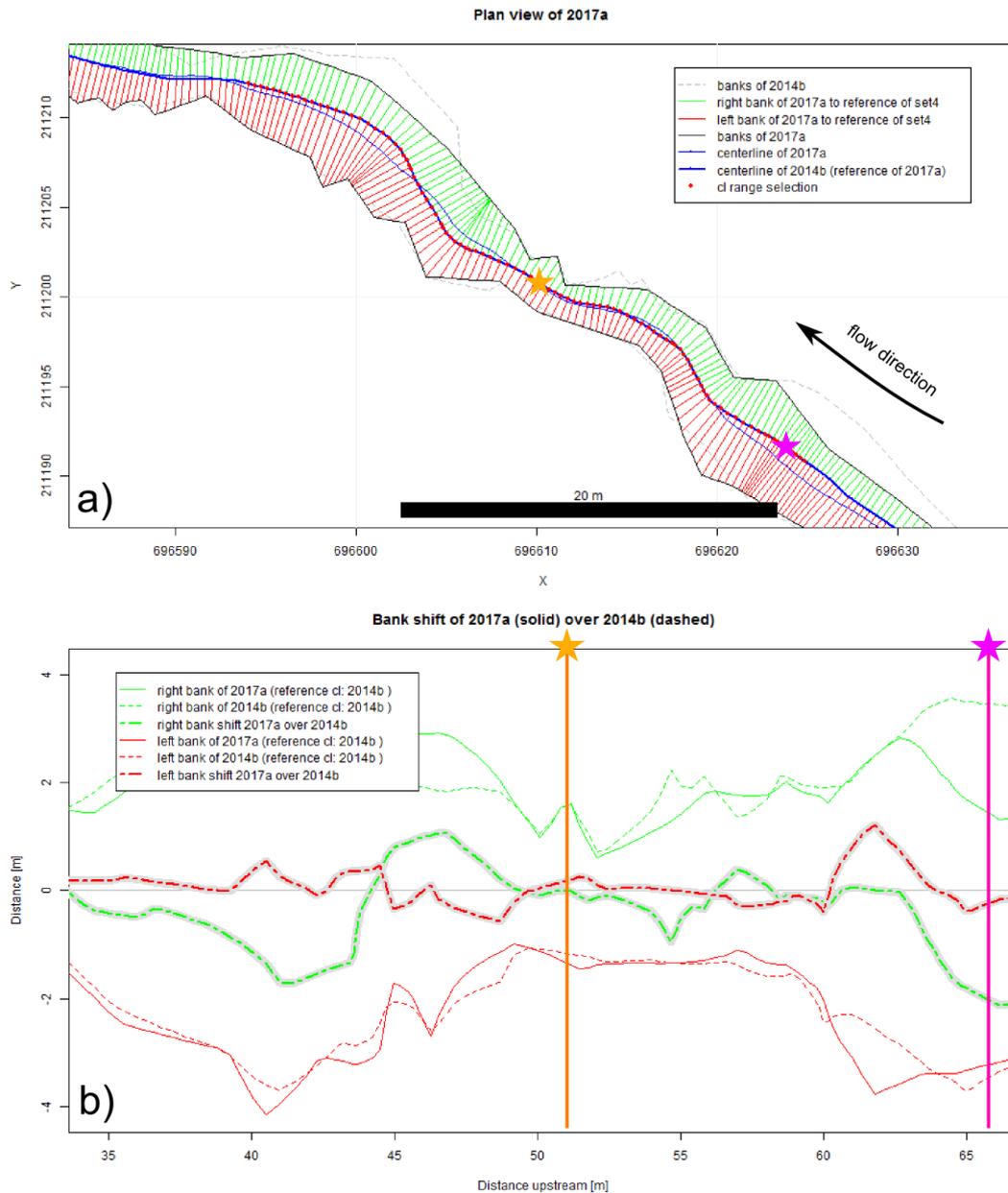


Figure 5: a) plan view of a short channel reach showing two channel surveys, 2014a (dashed channel outline) and 2017a (solid channel outline). A centerline is calculated for both, but due to an enabled reference mode, the centerline of 2014a is used for both surveys. This allows for the calculation of bank shift in b). The two stars mark to random locations to compare the calculated metrics to each other.

289 which is determined by a center coordinate (x,y coordinate) and the range on the x and y axes (zoom
 290 length). The zoom length is given via the function parameter `zoom.length`, or – if left empty – is
 291 taken from the global parameter `cmgo.objparplot.zoom.extent.length` (140 m by default). Multiple
 292 ways exists to determine the center coordinate: via pre-defined plot extent, via centerline point
 293 index, or directly by x/y coordinates. Pre-defined plot extents allow for quickly accessing
 294 frequently considered reaches of the stream and are stored in the parameter list (see Codebox 5).
 295 The list contains named vectors, each with one x and one y coordinate. To apply a pre-defined

296 extent the name of the vector has to be passed to the plot function as in `CM.plotPlanView(cmgo.obj,`
 297 `extent="extent_name")`. Another way of specifying the plot region is via a centerline point index, for
 298 example `CM.plotPlanView(cmgo.obj, cl=268)`. This method guarantees that the plot gets centered on
 299 the channel. To find out the index of a desired centerline point, centerline text labels can be enabled
 300 with `cmgo.objparplot.planview.cl.tx = TRUE`. Finally, the plot center coordinate can be given
 301 directly by specifying either x- or y-coordinate or both. If either x- or y-coordinate is provided, the
 302 plot centers at that coordinate and the corresponding coordinate will be determined automatically
 303 by checking where the centerline crosses this coordinate (if it crosses the coordinate multiple times,
 304 the minimum is taken). If both x and y coordinates are provided, the plot centers at these
 305 coordinates.

```

plot.zoom.extents = list(      # presets (customizable list) of plot regions
  e1 = c(400480, 3103130),    # plot region definition e1 with x/y center coordinate
  e2 = c(399445, 3096220),
  e3 = c(401623, 3105925),
  all = NULL
)

```

306
 307 **Codebox 5: definition of pre-defined plot extents that allow to quickly plot frequently used map regions. The names, here “e1”,**
 308 **“e2”, “e3”, contain a vector of two elements, the x and y coordinates where the plot is centered at. To plot a pre-defined region**
 309 **call for example `CM.plotPlanView(cmgo.obj, extent="e2")`.**

310 A plot of the width of the whole channel (default) or for a portion (via `cl` argument) can be created
 311 with `CM.plotWidth()`. Two data sets with the same reference centerline can also be compared. The `cl`
 312 argument accepts the range of centerline points to be plotted, if `NULL` (default) the full channel length
 313 is plotted. If a vector of two elements is provided (e.g. `c(200, 500)`), this `cl` range is plotted. If a
 314 string is provided (e.g. "cl1"), the range defined in `cmgo.objparplot.cl.ranges$cl1` is plotted.
 315 Alternatively to the range of centerline indices, a range of centerline lengths can be provided with
 316 argument `d`. If a single value (e.g. `500`) is given 50m around this distance is plotted. If a vector with
 317 two elements is given (e.g. `c(280, 620)`) this distance range is plotted.

318 The third plot function creates a plot of the bank shift (bank erosion and aggradation). This plot is
 319 only available when using multiple channel observations in the reference centerline mode (see
 320 section 5.1). The arguments of the function regarding the definition of the plot region is the same
 321 as of the function `CM.plotWidth()`.

322 In addition to the plotting, the results can be written to output files and to an R workspace file with
 323 the function `CM.writeData()`. The outputs written by the function depend on the settings in the
 324 parameter object. If `cmgo.objparworkspace.write = TRUE` (default is `FALSE`) a workspace file is
 325 written containing the global data object. The filename is defined in
 326 `cmgo.objparworkspace.filename`. Further, ASCII tables can be written containing the centerline

327 geometry and the calculated metrics. If `cmgo.objparoutput.write = TRUE` (default is `FALSE`) an output
328 file for each data set is written to the output folder specified in `cmgo.objparoutput.dir`. The file
329 names are the same as the input filenames with the prefixes `cl_*` and `metrics_*`. All parameters
330 regarding the output generation can be accessed with `?CM.par` executed in the R console or can be
331 found in the SM I.

332 5. Temporal analysis of multiple surveys

333 The program can perform analyses on time series of channel shapes. To do this, multiple input files
334 have to be stored in the input directory (see section 4.1). A data set for each file will be created in
335 global data object, mapped to the sub lists “set1”, “set2”, etc. (see Codebox 1). The program
336 automatically iterates over all data sets, processing each set separately. The order of the data sets is
337 determined by the filenames. Thus, the files need to be named according to their temporal
338 progression, e.g. “channelsurvey_2017.csv”, “channelsurvey_2018.csv”, etc. The mapping of the
339 filenames to data sets is printed to the console and stored in each data set under
340 `cmgo.obj$data[[set]]$filename`.

341 5.1. REFERENCE CENTERLINE

342 The channel metrics are calculated based on the centerline, which exists for every river bed
343 geometry. When there are multiple temporal surveys of a river geometry, a centerline for each data
344 set exists. Multiple centerlines prevent a direct comparison of the channel metrics as they can be
345 seen as individual channels. Thus, for temporal comparisons of the channel metrics, two modes
346 exist. Metrics are either calculated for each channel geometry individually. In this mode, the
347 channel metrics are the most accurate representation for that channel observation, for example
348 channel width is most accurately measured, but do not allow for a direct comparison of consecutive
349 surveys. In a second approach, a reference centerline for all metrics calculations can be determined.
350 In this approach, all metrics for the various bank surveys are calculated based on the centerline of
351 the data set defined in `cmgo.objparcenterline.reference` (default “set1”). This mode must be
352 enabled manually (see Codebox 6). This option should only be used if the bank surveys differ only
353 slightly. If there is profound channel migration or a fundamental change in the bed geometry, the
354 calculated channel metrics might not be representative (shown in Figure 6). To compare channel
355 geometries differing like that we recommend to calculate the metrics based on individual
356 centerlines and develop a proper spatial projection for temporal comparisons.

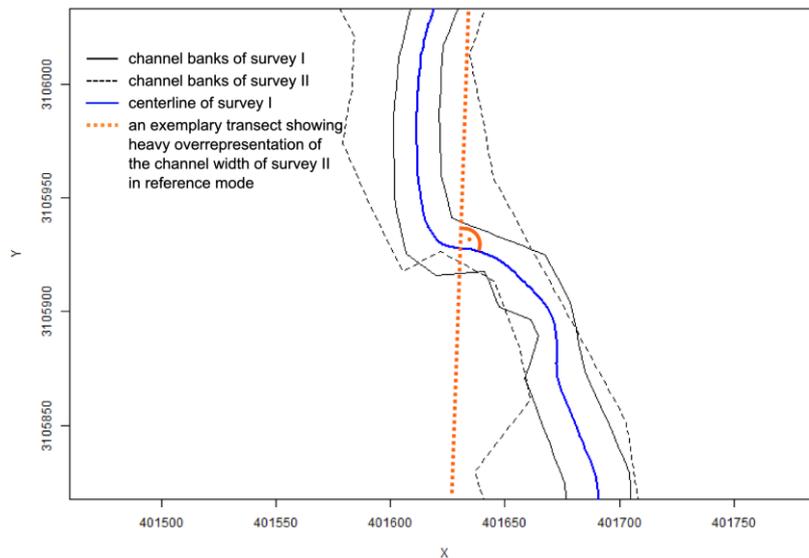


Figure 6: two consecutive channel geometries (survey I and II) with a profound reorganization of the channel bed. In the reference mode a centerline of one survey is used to build transects. Here, using the centerline of the first survey (blue line) as a reference is not suitable to capture the channel width correctly for the second survey (dashed line) as the exemplary transect (dashed orange line) suggests.

```
cmgo.obj$par$centerline.use.reference = TRUE
cmgo.obj$par$centerline.reference    = "set1"
```

Codebox 6: the parameters to enable the reference mode for channel metrics calculations (only necessary for time series analyses).

6. Technical fails and how to prevent them

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There are certain geometrical cases in which the algorithm can fail with the default parametrization. To prevent this, a customized parametrization of the model is required. The program prints notifications to the console during runtime if the generation of the centerline fails and offers solutions to overcome the issue. The main reason for failure occurs if the resolution of channel bank points (controlled via `cmgo.objparbank.interpolate.max.dist`) is relatively low compared to the channel width. In tests, a `cmgo.objparbank.interpolate.max.dist` less than the average channel width was usually appropriate. Otherwise, the desired centerline segments produced by the Voronoi polygonization can protrude the bank polygon (Figure 7a) and thus do not pass the initial filter of the centerline calculation (see section 0), since this filter mechanism first checks for segments that lie fully within the channel polygon. This creates a gap in the centerline, which results in an endless loop during the filtering for dead ends. Thus, if problems with the calculation of the centerline arise, an increase of the spatial resolution of bank points via `cmgo.objparbank.interpolate.max.dist` is advised to naturally smooth the centerline segments (see Figure 7b).

371 Another problem can arise from an unsuitable setting during the calculation of transects. If the
 372 channel bed exhibits a sharp curvature a misinterpretation of the channel width can result (see
 373 Figure 8). In that case, one of the red transects does not touch the left bank of the channel properly,
 374 thus leading to an overestimated channel width at this location. To prevent this, the span of the
 375 transect calculation can be increased. The results have to be checked visually by using one of the
 376 plotting functions of the package.

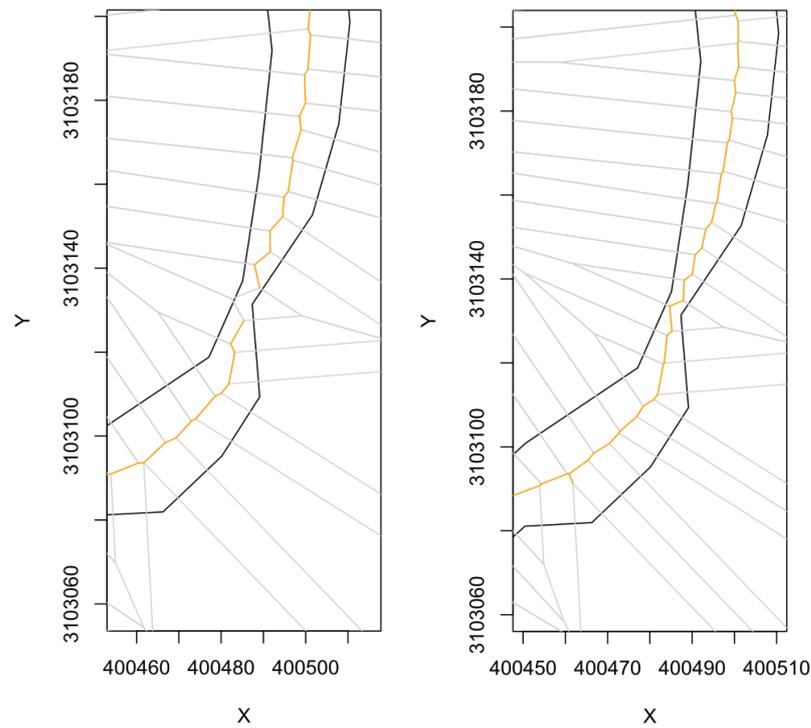
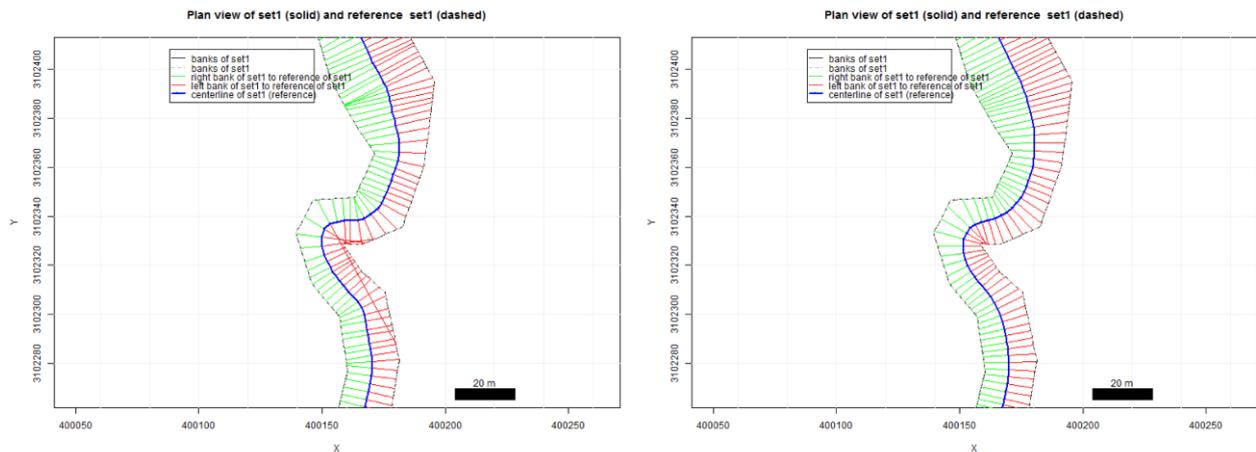


Figure 7: a gap in the centerline occurs when the spacing of the bank points is too high compared to the channel width (left) which can be fixed by previously increasing the resolution of the bank points (right).



377
 378 **Figure 8: left: the transects (perpendiculars to the centerline) do not intersect with banks properly, thus the channel width is**
 379 **overrepresented. Right: an increased transect span fixes the problem and channel width is now identified correctly.**

380 7. How to use the program: step by step instructions

381 *cmgo* can be used even without comprehensive R knowledge and the following instructions do not
382 require preparatory measures other than an installed R environment (Yan et al. 2011). Once the R
383 console is started, installation of the *cmgo* package is done with the `install.packages()` function
384 (Codebox 7).

385 To quickly get started with *cmgo*, we provide four demo data sets. Using these data sets the
386 following examples demonstrate the main functions of the package, but, more importantly, allow
387 to investigate the proper data structure of the global data object. This is of particular importance
388 when trouble shooting failures with custom input data.

389 The general execution sequence includes initialization, processing and reviewing the results, with
390 a standard execution sequence shown in Codebox 8. To switch from demo data to custom data,
391 input files have to be placed in the specified input folder (“./input” by default) and `CM.ini()` has to
392 be called without any arguments. Since the file format of the custom input files can differ from the
393 expected default format, all program parameters regarding the data reading should be considered.

394 A list of all parameters available can be accessed with `?CM.par` executed in the R console or can be
395 found in the SM I. To change a parameter, the new parameter value is assigned directly within the
396 global data object (e.g. `cmgo.objparinput.dir = "./input"`).

397 The plotting functions include a map-like plan view plot (`CM.plotPlanView()`), a line chart with the
398 channel width (`CM.plotWidth()`) and, if available, a plot of the bank retreat (`CM.plotMetrics()`). The
399 latter is only available in the reference centerline mode (see section 5.1).

```
# installation of dependencies (required only once)
install.packages(c("spatstat", "zoo", "sp", "stringr"))

# installation (required only once)
install.packages("cmgo", repos="http://code.backtosquareone.de", type="source")

# include the package (required for every start of an R session)
library(cmgo)
```

400

401 **Codebox 7: installation and embedding of the package in R**

```
# initialization: load data and parameters
cmgo.obj = CM.ini("demo") # check the data structure with str(cmgo.obj)

# processing
cmgo.obj = CM.generatePolygon(cmgo.obj)
cmgo.obj = CM.calculateCenterline(cmgo.obj)
cmgo.obj = CM.processCenterline(cmgo.obj)

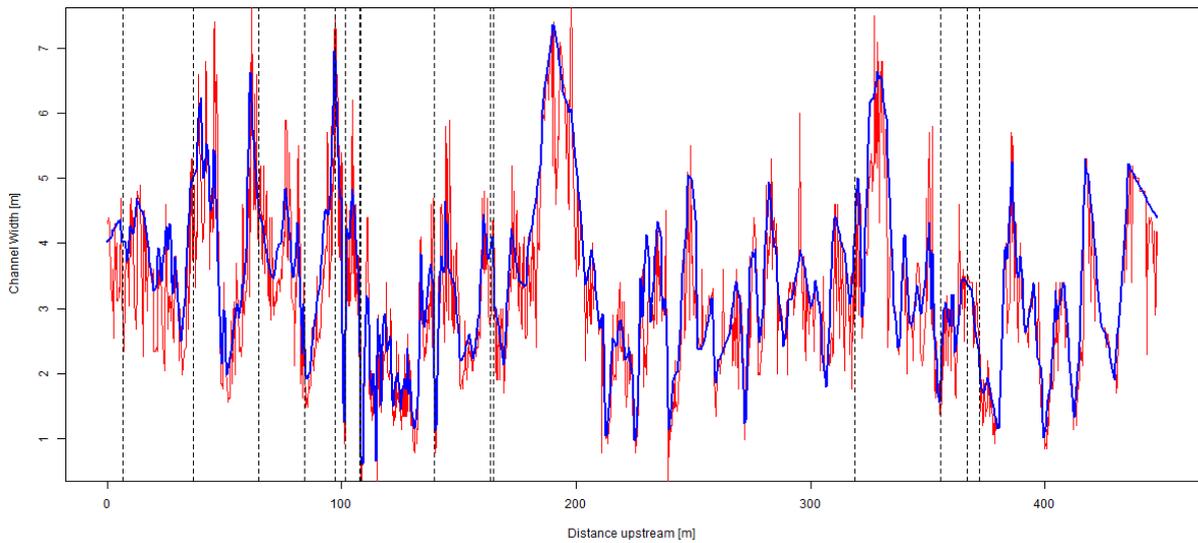
# view results
CM.plotPlanView(cmgo.obj) # plot a map with pre-defined extent
CM.plotWidth(cmgo.obj) # plot the channel width in downstream direction
CM.plotMetrics(cmgo.obj) # plot a comparison of bank profiles
```

402

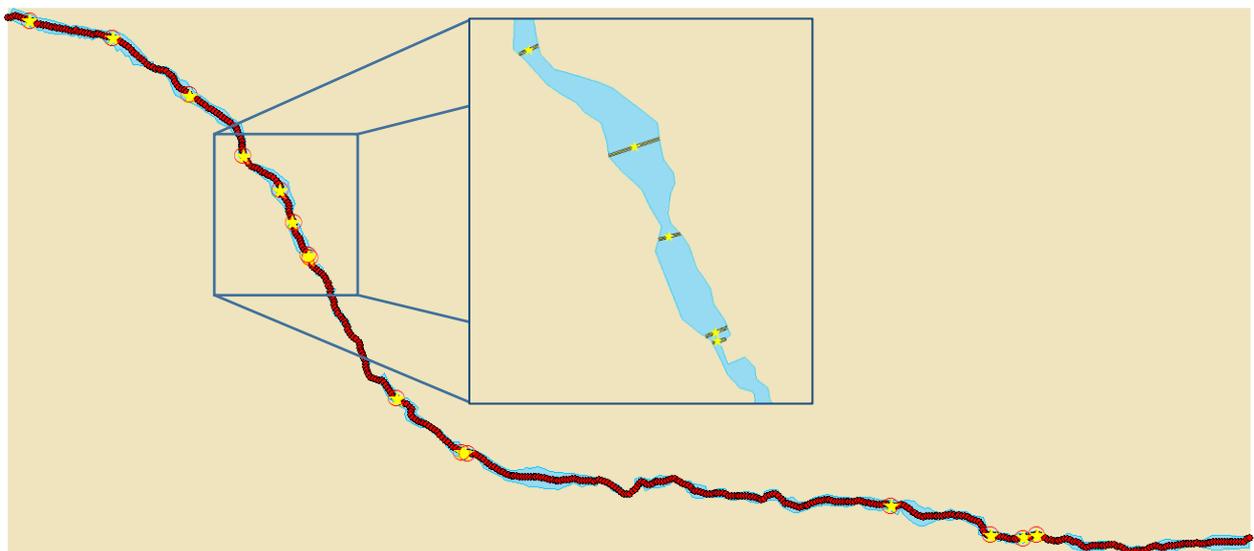
403 **Codebox 8: minimal example script to run *cmgo* with demo data set.**

404 **8. Evaluation of the data quality**

405 We evaluated the quality of the derived channel width by cmgo to manually measured data and to
406 the best documented and versatile product of our literature review RivMap. First, we compared the
407 evolution of the channel width derived by the two automated products showing that there is a
408 general agreement (Figure 9). We picked 15 locations randomly, marked with the dashed vertical
409 lines, where we assessed the channel width manually (Figure 10). In a GIS we measured channel
410 width manually at these 15 locations on a “best guess” approach.



411 **Figure 9: channel width as observed by cmgo (blue line) and RivMap (red line) for 1506 locations along a 449 m reach of a**
412 **natural channel in upstream direction. The vertical dashed lines mark our points where we investigated the width manually**
413 **next.**
414



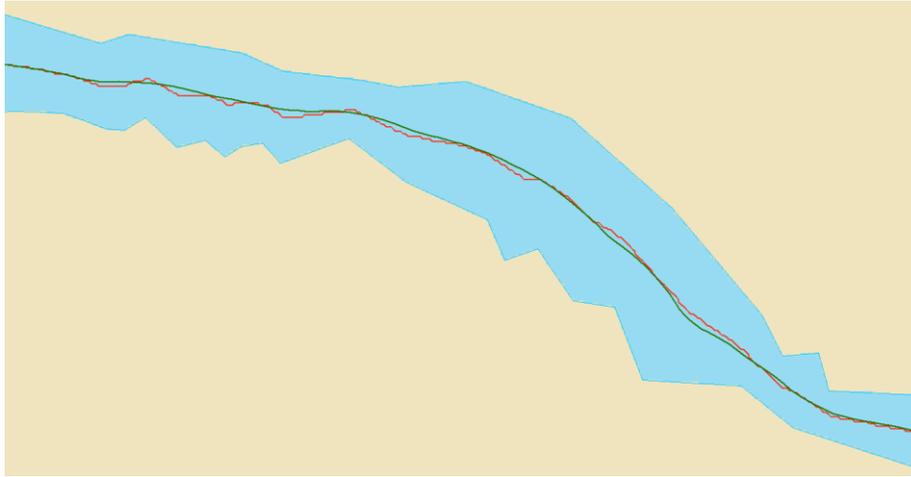
415 **Figure 10: 15 random locations where we evaluated the width manually and compared to the width of the automated products.**
416

417 The channel width at the transects is generally well captured by the automated products (Table 3)
418 as the mean errors are relatively low compared to the absolute width. However, compared to the
419 manually derived average width of 3.49 m the average width of all transects deviates only -0.07 m
420 for *cmgo* while it deviates -0.42 m for *RivMap*. Thus, *cmgo* performs generally better in deriving
421 the channel width for the test channel reach and *RivMap* seems to consistently underestimate the
422 channel width. This is also expressed in the smaller standard deviation of the differences which is
423 0.098 m for *cmgo* and 0.736 m for *RivMap*. The large scatter can also be observed in Fig. 9.
424 Compared to the error of the in-situ measurements of the channel banks with a total station (1 cm)
425 the precision of the channel width calculations by *cmgo* is within the same order of magnitude while
426 it is an order of magnitude larger for *RivMap*.

427 The channel centerlines of the two products differ in length. While the centerline of *cmgo* has a
428 length of 449 m along the river reach, the centerline of *RivMap* has a length of 588 m (30% longer).
429 Looking at the shape of the centerlines (Figure 11) we argue that the centerline of *cmgo* better
430 represents the channel in terms of large scale phenomena. It may for example be more useful for
431 reach-averaged calculations of bankfull flow. The centerline of *RivMap* contains a stronger signal
432 of the micro topography of the banks due to the way the centerline is created (eroding banks). The
433 difference in length also has an influence on slope calculations which will be lower for *RivMap*.

Transect [No.]	Manual approach [m]	CMgo width [m]	CMgo difference to manual [m]	RivMap width [m]	RivMap difference to manual [m]
1	4.01	4.02	0.01	2.83	-1.18
2	5.01	5.02	0.00	3.75	-1.27
3	4.57	4.55	-0.01	4.03	-0.54
4	2.66	2.59	-0.07	2.60	-0.06
5	6.79	6.83	0.04	5.37	-1.41
6	2.82	2.66	-0.15	2.12	-0.70
7	3.02	2.97	-0.06	2.55	-0.48
8	1.76	1.67	-0.09	2.60	0.84
9	2.27	1.93	-0.34	2.60	0.33
10	3.90	3.91	0.01	2.83	-1.07
11	3.82	3.66	-0.17	4.40	0.58
12	4.19	4.14	-0.05	3.04	-1.15
13	2.04	1.89	-0.15	1.34	-0.70
14	3.37	3.37	0.00	3.50	0.13
15	2.14	2.11	-0.03	2.50	0.36
avg.	3.49	3.42	-0.07	3.07	-0.42
st. dev.	1.340	1.399	0.098	0.997	0.736

Table 3: channel width at 15 randomly selected locations along a natural channel. The width was identified manually, by CMgo and by RivMap. Differences of the width from the automated products were compared to the manual approach.



434

435 **Figure 11: the two different centerlines of the products cmgo (green line) and RivMap (red line) reveal differences in the shape**
 436 **that influence also the channel length**

437 9. Concluding remarks

438 The presented package *cmgo* offers a stand-alone solution to calculate channel metrics in an
 439 objective and reproducible manner. At this, *cmgo* allows for close look into the interior of the
 440 processing. All intermediate results are accessible and comprehensible. Problems that arise for
 441 complex geometries can be overcome due to the high degree of parametrization. *cmgo* qualifies for
 442 a highly accurate tool suited to analyze especially complex channel geometries. However, if
 443 complex geometries should be compared to each other, for example when analyzing the evolution
 444 of meandering channels, our product does not offer the ideal solution due to the style *cmgo* treats
 445 the reference of the channels. Thus, our product should be the tool of choice if precise
 446 measurements – both in location and quantity – are required and if geometrical and other spatial
 447 data should be statistically analyzed. However, when large time series of meandering rivers are the
 448 main purpose of the effort, other products, as for example the Channel Migration Toolbox, are more
 449 suitable.

450 Since *cmgo* does not come with graphical user interface only static map views of the channel can
 451 be obtained by scripting them. *cmgo* offers various plotting functions to do this which allow for
 452 predictable and reproducible plot. The downside of this approach is that plots are naturally not
 453 interactive which is the case for GIS applications. For people who prefer this functionality an export
 454 of the intermediate and end results to GIS is recommended.

455 The only requirement for running *cmgo* is an installed environment of the open source framework
 456 R. Thus, the prerequisites are narrowed down to a minimum to facilitate an easy integration and
 457 wide a distribution for scientific or practical use. The license under which the package is provided
 458 allows modifications to the source code. The nature of R packages determines the organization of

459 the source code in functions. This encapsulation comes at the cost of a sometimes untransparent
460 architecture making it difficult to modify or understand the code. Thus, for advanced users, who
461 desire a more flexible way of interacting with the algorithm, we refer to the raw source codes at
462 GitHub (<https://github.com/AntoniusGolly/cmgo>).

463 10. Code and Data availability

464 All codes and demo data are available at <https://github.com/AntoniusGolly/cmgo>.

465 11. Team list

466 Antonius Golly (Programming, Manuscript), Jens Turowski (Manuscript)

467 12. Competing interests

468 The authors declare that they have no conflict of interests.

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