Answers on the review for NPG-2016-80 by M. Kuznetsova, E. Pasternak and A. Dyskin, “Analysis of Wave Propagation in a Discrete Chain of Bilinear Oscillators”

The authors are grateful for the perusal and important suggestions provided by the reviewer. All the suggestions have been taken into account and the manuscript has been changed accordingly.

Changes

1. Please emphasize the differences with the paper [Gavrilov S.N. and Herman G.C., 2012].

   Response
   We added the following sentences in the last paragraph of the introduction:
   “In order to compare the chain of bilinear oscillators with its homogenised counterpart, we also considered a continuous 1D bimodular rod and developed a solution for its wave equation. In doing so, we will not restrict ourselves to small difference in stiffnesses, thus providing a more general analysis than the ones presented in (Naugonlykh and Ostovsky, 1998), (Gavrilov and Herman, 2012).”

2. What is the goal of this research?
   Response
   We added the following sentence in the next-to-last paragraph of the introduction:
   “The purpose of the present work is to study the response of a discrete system of bilinear oscillators loaded by an external harmonic force, especially for the case of the large difference between spring stiffnesses in tension and compression.”

3. It is necessary to note that in the case “tension – compression” the analytical solution exists for the stiffness ratio a<<1, when the order of the Eq. (11) can be reduced and a solution with shock front exists, see [Naugolnykh, K., & Ostrovsky, L. (1998). *Nonlinear wave processes in acoustics*. Cambridge University Press]. In this connection, please refine the peculiarity of the presented study.
   Response
   It should be noted that approximate analytical and numerical results for 1D bimodular rod are presented in (Naugonlykh and Ostovsky, 1998). However, they were obtained for a considerable limitation on the stiffness ratio being close to $\cancel{\ll}$1, whereas we purposefully consider a case of the large difference between moduli in tension and compression as the most representative example of collision between tensile and compressive wave fronts and conservation of energy and the same time.
   We added the following sentence in the last paragraph of the introduction:
   “In doing so, we will not restrict ourselves to small difference in stiffnesses, thus providing a more general analysis than the ones presented in (Naugonlykh and Ostovsky, 1998), (Gavrilov and Herman, 2012).”

4. Please estimate the product of the characteristic wave number and the length of the spring.
   Response
   We added the following information in the beginning of Sect. 6:
   In order to ensure whether the discrete chain with the given parameters can be considered as a continuum, let us estimate the dimensionless wave length λ

$$\lambda = \frac{c_t \pi}{\omega} \approx 10.26$$

The obtained wave length λ is much greater than the spring length l_s, assumed to be equal to 1 (see Table 1), which is why the continuum approximation becomes possible.

5. What methods of numerical simulations were used?
Response
We added the following sentence in Sect. 4:
"An explicit Runge-Kutta method with the time step $\Delta t = 10^{-3}$ is used for solving the system of N bilinear ODEs (6) in Sect. 4."
We also have the following information in Sect. 6:
"Numerical results are obtained by solving Eq. (12) using the explicit central difference scheme. To match the results obtained for the discrete chain of bilinear oscillators, spatial and time discretisation is chosen to be the same ($\Delta x = 1$ and $\Delta t = 10^{-3}$, respectively) and all other parameters being used from the Table 1."

Typographical mistakes

1. What is the correct notation for masses *m* or *M*?
Response
Corrected in Sect. 2:
"We consider an infinite chain of masses and bilinear springs, where masses M are supposed to be identical, springs have the length L_s and the stiffness described in the following formula"

2. The same for ω and Ω.
Response
Corrected in Sect. 2:
"Here and in what follows, consider harmonic external loading of the type $F(T) = F_0 \sin(\Omega T)$ where F_0 denotes any multiplier in front of the harmonic function and Ω denotes the external excitation frequency."

"ω is the dimensionless excitation frequency $\omega = \frac{\Omega}{\Omega_0}$."

3. Is the frequency of the applied force ω in the Table dimensionless?
Response
Yes. We changed its definition in Sect. 2 and altered the Table in Sect. 3:
"ω is the dimensionless excitation frequency $\omega = \frac{\Omega}{\Omega_0}$."
"All the numerical results presented in the paper are obtained for the following dimensionless parameters:"

4. One Heaviside function is used in the Eq. (8) instead of difference of two Heaviside functions.
Response
Corrected in Sect. 4 and Sect.5:

$$f(t) = \pm f_0 H(t) H\left(\frac{2\pi}{\omega} - t\right) \sin(\omega t), \quad f(t) = \pm f_0 H(t) \sin(\omega t).$$