



1	Appraisal on inversion algorithm techniques in 2D electrical resistivity tomography
2	survey data for poised mapping of subsurface features
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6	Abstract:
7	Analysis of non-uniqueness model in resistivity imaging data is vital in inaugurating the
8	consistency of models. Nevertheless, such analysis is moderately unusual in resistivity
9	imaging data set. Electrical resistivity tomography (ERT) technique is being constantly used
<mark>10</mark>	in many scientific areas including engineering, environmental and archaeological survey,
11	Primarily, the inversion algorithm techniques are employed on synthetic model data set with
<mark>12</mark>	and without some random Gaussian noise, and its validity is tested by filed data set. The
13	study was conducted in the premises of Central Institute of Mining and Fuel Research
14	(CIMFR), Dhanbad by laying an ERT profile of 480 m length with 5 m electrode spacing
15	using Syscal Pro (Iris instrument) resistivity meter. Two standard arrays were used in this
16	study namely Wenner-Schlumberger and dipole-dipole array. The data set was mixed to a
<mark>17</mark>	single array to achieve better resolution and enhanced clarification. On processing data by
<mark>18</mark>	Prosys-II software, it was exported in Res2Dinv software for inversion. In this context, data
19	was inverted by different algorithm techniques i.e. least square (L2-norm) and robust
20	inversion (L ₁ -norm). Exemplary results related to the heterogeneity of the resistivity structure
21	within the high and low resistivity anomaly were obtained by robust inversion method. The
22	obtained results are in broad agreement with the simulation model.
23	Keywords: ERT, Least square, Robust inversion, CSIR-CIMFR, Existing structures
24	





26 **1.0 Introduction:**

27	A significance of ground based geophysical technique is related to as much information as
<mark>28</mark>	imaginable on subsurface existing structures and composition of materials which remain in
<mark>29</mark>	the subsoil. All surface geophysical techniques are function of physical properties of the earth
30	materials. Changes in subsurface properties such as porosity, permeability, density, saturation
31	of water etc. may be distinguished by geophysical survey like gravity, seismic, and electrical
32	methods (e.g., Ezersky 2008; Keller and Frischknecht 1996). Currently, it is common to
<mark>33</mark>	apply geophysical techniques to environmental, engineering and mining related problems at
<mark>34</mark>	shallow depths and it is valid solution for target identification at complex subsurface
<mark>35</mark>	structures.
<mark>36</mark>	A non-invasive surface geophysical technique such as Electrical Resistivity Tomography
<mark>37</mark>	(ERT) belongs to the family of the most applied geophysical methods in an extensive
<mark>38</mark>	spectrum of mapping of near surface problems and environmental studies (e.g., Singh et al.,
<mark>39</mark>	2004; Dahlin and Zhou 2006; Chandra et al., 2008; Kumar 2012; Singh 2013b; Bharti et al.,
<mark>40</mark>	2016a,b; Bharti et al., 2019). This technique has become the most routinely used geoelectrical
<mark>41</mark>	application for delineating the complex geological features of subsurface of the earth due to
<mark>42</mark>	its comparative effortlessness and time effectivity. A better understanding of the subsurface
<mark>43</mark>	geoelectrical structures in hard rocky terrain can be achieved by this technique. Electrical
<mark>44</mark>	properties of the earth mass at shallow depth can be obtained by 2D ERT technique in both
<mark>45</mark>	vertical and horizontal orientations, which helps in notching up of status of strata in
<mark>46</mark>	qualitative and quantitative forms.
47	The geoelectric distinction between dissimilar natures of earth constituents is a reasonable
<mark>48</mark>	means to classify different material characteristics, which has been allocated to the degree of
<mark>49</mark>	weathering, moisture content and mineralogical composition of such earth material. A cell-

50 based inversion technique is usually implemented for effectively prototypical complicated





structures along an uninformed resistivity spreading in subsurface of the earth. Therefore, this
procedure makes numerous rectangular cells with fixed positions and sizes by division of
subsurface block.

54 A few researchers have done work over comparison of inversion techniques in 2D resistivity data sets. For example, the work done by Loke (2003), encompasses analysis on smooth and 55 56 blocky inversion methods in 2D resistivity survey. According to them, better results are 57 obtained by smooth inversion method in which change in resistivity is gradual; on the other hand outcomes of blocky inversion method gives significant results for sharp boundaries. The 58 59 study was carried out over karstic structures by Hamdan and Vafidis (2009), by inversion techniques for eminent image of resistivity. Three different inversion methods i.e. combined, 60 61 smoothness constrained and robust inversion were adopted on real data set and results were 62 compared and also combined inversion of two standard configurations namely, Wenner-63 Schlumberger and dipole-dipole was conducted to obtain the highest reliability of the 2D resistivity section. 64

65 The CSIR-CIMFR campus situated in Dhanbad area, India. Dhanbad, the coal capital of country, lies in the mid-eastern part of Jharkhand state. Dhanbad district is evaluated as dry 66 because of deficiency of immense rivers and high temperature. The district is related to small 67 scale of ponds and two big dams which are good medium to recharge groundwater. 68 69 Therefore, groundwater arises in this zone below unconfined state in the weathered formations at low depths in utmost of the lithological components in the Achaeans and nearly 70 all the lithological components in the Gondwana formation. Groundwater arises below 71 72 confined to semi confined state where the fractures are deep seated and are unrelated with the top weathered formation (e.g., Kumar 2018). 73





76 **2.0 Methodology:**

In general, inversion procedure is involved to renovate the real circulation of acquired 77 78 resistivity data sets as the latter does not deliver anticipated facts. The present study was 79 conducted over the premises of Central Institute of Mining and Fuel Research, Dhanbad, India as shown in Fig. 1. Keeping in view the literature review in background, the scope of 80 81 study encases analysis of two different inversion methods, namely, least square (L_2 -norm) 82 and robust inversion (L1-norm) in 2D resistivity data set for mapping of complex subsurface existing structures. The idea of multiple inversion techniques could be used for evaluating the 83 84 superiority of true 2D resistivity models. Inversion technique is a procedure to create a model 85 that clarifies a set of measurements. It is related to make direct assumptions about the earth 86 from DC resistivity measurements due to the contests of envisaging large data sets (e.g., Loke 87 et al., 2003).

88 2.1 Synthetic Model:

Initially, the comparison of two different algorithm techniques i.e. least square and robust inversion in 2D electrical resistivity data set to map the complex subsurface existing structures through forward modelling, considered for better interpretation of field data set using RES2DMOD software package (e.g., Loke and Barker, 1996).

93 In this context, the model was consisted of four homogeneous layers i.e. (i) soil/alluvium layer, (ii) semi weathered rock layer, (iii) hard weathered rock layer and (iv) bed rock/ 94 basement rock layer where their apparent resistivity values are of 100 Ω m, 300 Ω m, 500 Ω m, 95 and 1000 Ω m with 64 equally spaced electrodes with 5m interval using finite difference 96 97 algorithm technique. Finite difference algorithm technique divides the model subsurface into a number of rectangular blocks (e.g., Loke et al., 2003). Two conductive body (resistivity 98 ranging 10 Ω m to 100 Ω m) and one resistive body (resistivity ranging 1000 Ω m to 2000 Ω m) 99 100 was incorporated in model set.





101 The simulated resistivity retorts of the section were initiated using Wenner-Schlumberger, 102 dipole-dipole and combined inversion of both arrays with and without some random Gaussian 103 noise added to validate field condition and get more representative results. The synthetic 104 apparent resistivity model data set was inverted by using RES2DINV software for producing 105 true resistivity variation of subsurface of the earth.

106 Figure 2 and 3 shows the obtained outcomes from Wenner-Schlumberger, dipole-dipole and combined inversion of both arrays using least square and robust inversion algorithm 107 techniques. Stimulatingly, all outcomes recover the anomaly locations through both inversion 108 109 techniques. However, in robust inversion technique was recognized both depth and 110 extensions of anomaly in all inverted resistivity models with greater resolution compared to 111 least square technique. It is also observed that the combined inversion of both arrays gives 112 the better results with high resolution compared to Wenner-Schlumberger and dipole-dipole 113 array. For example, Dahlin and Zhou (2004), reported that the imaging with combined arrays generates models similar to the preferable observation model among the specific array. 114

115 2.2 Smooth-constrained least-squares technique:

This technique usually uses the form of regularised least-squares optimization method in the
smooth-constrained or L₂-norm. The mathematical expression of this technique (e.g.,
deGroot-Hedlin and Constable 1990; Ellis and oldenburg 1994) is expressed as:

119
$$(J_i^T J_i + \lambda_i W^T W) \Delta r_i = J_i^T g_i - \lambda_i W^T W r_{i-1}$$
(1)

120 Where, $g_i = data$ misfit vector,

121
$$\Delta r_i$$
 = change in the model parameters for the ith iteration,

122 W= roughness filter,

123 $\lambda =$ damping factor,

124 r_{i-1} = model parameters vector for the previous iteration and

125 J = Jacobian matrix of partial derivatives.





- Roughness is filtered by first-order finite difference operator (e.g., deGroot-Hedlin andConstable 1990). The equation (1) helps in minimising the sum-of-squares of the data misfit
- 127 Constable 1990). The equation (1) helps in minimising the sum-of-squares of the data mi
- 128 and sum-of-squares of the model roughness.
- 129 Smooth-constrained least-squares technique reduces the sum of squares of the spatial changes
- in the model resistivity and the data misfit. Optimal results are obtained for geologicallysmooth variation subsurface (e.g., Barker 1992). However, it shows spread boundaries for
- 132 sharp transition like igneous dyke.

133 **2.3 Robust or blocky inversion technique:**

The cumulated absolute value of spatial changes in resistivity model can be reduced by
Robust inversion technique. It is also known as L₁-norm measure of the data misfit (e.g.,
Claerbout and muir 1973). The mathematical formulations used by L₁-norm optimisation
method is

138
$$(J_i^T R_d J_i + \lambda_i W^T R_m W) \Delta r_i = J_i^T R_d g_i - \lambda_i W^T R_m W r_{i-1}$$
(2)

139 Where, R_d and R_m = weighting matrices

140 Constant resistivity values of each part are produced on application of L₁-norm to model
141 roughness filter (e.g., Farquharson and Oldenburg 1998). Sharp boundary separation is also
142 obtained by this technique.

143 **3.0 Discussions:**

2D ERT section of profile AA' was generated by the configurations of Wenner-Schlumberger, dipole-dipole and combined inversion of both arrays for the length of 480 m with electrode interval of 5m using Syscal Pro (Iris instrument) resistivity meter with 96 electrodes (Fig.2). Least square and robust inversion technique was adopted for analysis of subsurface existing geological formation using Res2Dinv handling software as shown in Figs. 3 & 4.





151 **3.1 Inverted geoelectrical section of Least square inversion of profile AA'**

The 2D geoelectric model of profiles AA' along with the least square inversion technique projected using Wenner-Schlumberger, dipole-dipole and combined inversion of both arrays are shown in Figures 3a, b & c respectively. The outcomes obtained by electrical resistivity tomography designate an extensive range of resistivity variation through the profile.

156 Topmost layer up to a depth of 10m consisting of soil/ alluvium having a resistivity of about 2 to 80 Ωm was considered for all ERT sections. Two water aquifers (L1Z2^{ws} & L2Z2^{ws}) 157 158 associated with fracture zones with relatively low resistivity of 2 to 12 Ω m at the surface 159 distance of about 130 m to 180 m and 280 m to 305m were delineated in 2D geoelectric section generated by Wenner-Schlumberger array (Fig.3a) and one water body (L2Z2^c) was 160 also identified in combined inversion of both arrays along the surface distance at about 280 m 161 to 305m (Fig.3c). Relatively high resistivity (230 to 608 Ω m) anomaly associated with 162 weather rock / fracture rock (WZ2^{dd} & WZ2^c) was identified along 2D ERT section of dipole-163 dipole and combined inversion of both arrays at reduced distance (RD) of 25 to 90 m (Fig.3b 164 165 & c). A high resistivity contrast of more than 1600 Ω m associated with bed rock/ hard rock (HZ3^{ws}, HZ3^{dd} & HZ3^c) was detected in all 2D resistivity sections (Wenner-Schlumberger, 166 dipole-dipole and combined inversion of both arrays) along a surface distance of 215 to 280 167 168 m.

169 **3.2 Inverted geoelectrical section of Robust inversion of profile AA'**

2D ERT inverse model of profiles AA' along with the robust inversion technique projected
using Wenner-Schlumberger, dipole-dipole and combined inversion of both arrays are shown
in Figures 4a, b & c respectively. Wide range in resistivity was observed by this technique
also.

Top layer consisting of soil/ alluvium was encountered up to depth of 10 m followed byWenner-Schlumberger, dipole-dipole and combined inversion of both arrays are shown in





Figures 4a, b & c respectively. A prominent signature of relatively low resistive (L2Z2^{ws}, 176 177 L2Z2^{dd}, L2Z2^c) water aquifer zone associated with fracture rock mass at RD of about 280 m to 305 m was identified with resistivity range of about 2 to 80 Ω m in all 2D ERT section 178 179 models. In addition, one water body (L2Z2^{ws}) was also identified in Wenner-Schlumberger array along the surface distance of about 280 m to 305m (Fig.4a). A signature of weather 180 rock / fracture rock (WZ1^{ws}, WZ1^{dd} & WZ1^c) was recognized along 2D ERT sections of 181 182 Wenner-Schlumberger, dipole-dipole and combined inversion of both arrays at RD of 25 to 110 m with moderately high resistivity range of 230 to 608 Ω m (Fig.4). The bed rock/ hard 183 rock (HZ3^{ws}, HZ3^{dd} & HZ3^c) with high resistivity signature of more than 1600 Ω m was 184 demarcated in all the 2D geoelectrical models of profile AA' projected by Wenner-185 Schlumberger, dipole-dipole and combined inversion of both arrays at the surface distance of 186 187 about 195 to 270 m.

The soil/ alluvium layer showed low resistivity up to 10 m depth by both the techniques. A signature of weather rock / fracture rock was delineated in least square inversion technique only for dipole-dipole and combined inversion of both arrays. However, in robust inversion technique this feature was visibly identified in all resistivity sections. The extension of water aquifer zone at greater depth associated with fracture rock mass was well demarcated by combined inversion of both arrays through L₁- norm in comparison to L₂-norm.

The outcomes generated of both synthetic and field conditions by inversion algorithm revealed that a combination of Wenner-Schlumberger and dipole- dipole array would provide maximum subsurface information and the optimal arrays sensitivity as this combination can encompass both strong signal/noise ratio and sensitivity to vertical and lateral changes. A prominent subsurface existing structure in geoelectrical sections by resistivity data sets could be assessed by comparing the outcomes of inversion techniques. This is vital particularly





200 where sudden resistivity changes like geologic interfaces characterized by variation in

- 201 lithology are anticipated.
- 202 **4.0 Conclusions:**

203 Initially, the synthetic data was generated using Res2Dmod software. Field situation was simulated through forward modelling. Two different algorithm techniques i.e. Least square 204 205 inversion and Robust inversion were studied in 2D electrical resistivity data set for mapping 206 of complex subsurface existing structures over a part of the CSIR-CIMFR campus using Wenner-Schlumberger, dipole- dipole and combined inversion of both arrays. Robust 207 208 inversion indicates an additional feature with combined inversion of both arrays compared to 209 L₂-norm and it has good convergence throughout the iteration process, enabling easy analysis. The extension of aquifer zone associated with fracture rock mass at greater depth 210 211 with high resolution was well demarcated by robust inversion indicates an additional feature 212 with combined inversion of both arrays. A complex subsurface existing structure in 213 geoelectrical sections by ERT data sets could be evaluated by comparing the consequences 214 from the two inversion schemes.

5.0 Data availability: Outcomes are in the form of images shown in Figs.1, 2, 3, 4, 5 and 6.

There is no data in addition.

217 **6.0 Team list:** Abhay Kumar Bharti, Amar Prakash and Krishna Kant Kumar Singh

218 **7.0 Author contribution:**

Abhay Kumar Bharti: Conducted field investigation, data interpretation and preparation ofmanuscript.

Amar Prakash: Contributed in enhancing data interpretation and elevation in manuscriptquality.

Krishna Kant Kumar Singh: Contributed in site selection for investigation and datainterpretation.





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286	Figure.3: Synthetic model outcomes (a) synthetic geological formation (b) inverted
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295	inversion technique
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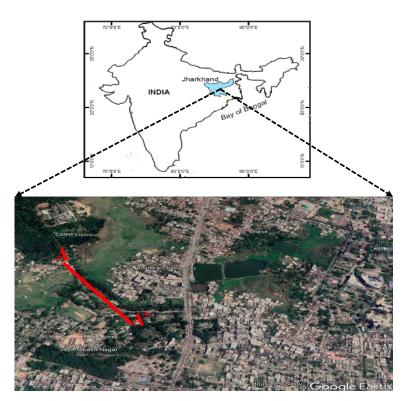


Figure.1: Location map of the study area © Google Earth





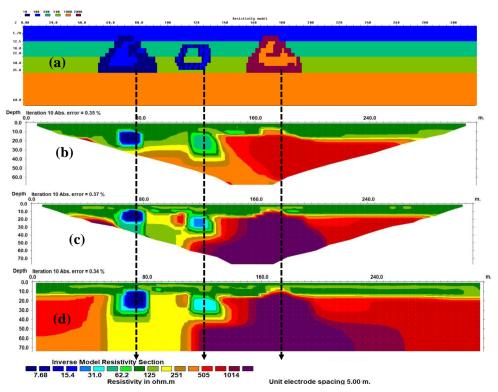


Figure.2: Synthetic model outcomes (a) synthetic geological formation (b) inverted resistivity model of Wenner–Schlumberger array (c) inverted resistivity model of dipole–dipole array and (d) combined inversion of both arrays with Least square inversion technique





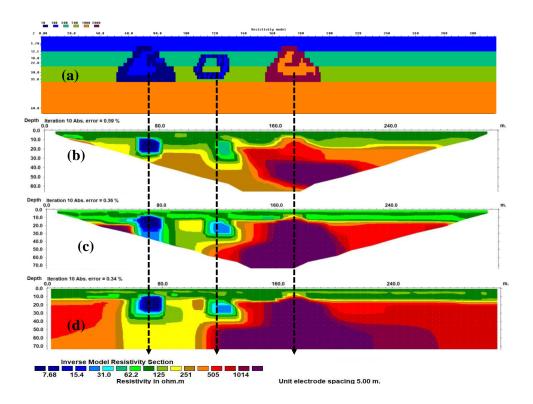


Figure.3: Synthetic model outcomes (a) synthetic geological formation (b) inverted resistivity model of Wenner–Schlumberger array (c) inverted resistivity model of dipole–dipole array and (d) combined inversion of both arrays with Robust inversion technique







Figure.4: SYSCAL Pro-96 (Iris Instrument) data-acquisition field setup using 96 electrodes





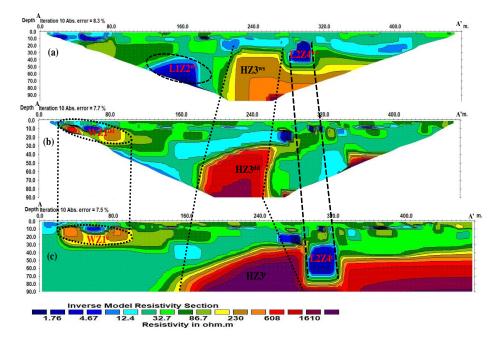


Figure.5: 2D ERT section along profile AA' over the study area: (a) Wenner–Schlumberger array, (b) dipole–dipole array and (c) combined inversion of both arrays with Least square inversion technique





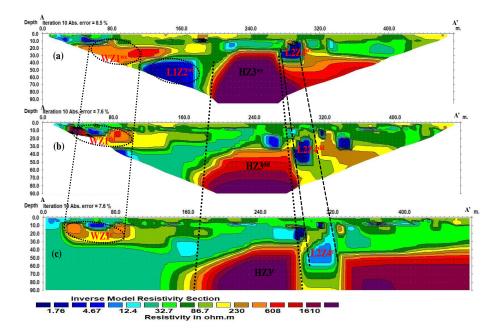


Figure.6: 2D ERT section along profile AA' over the study area: (a) Wenner–Schlumberger array, (b) dipole–dipole array and (c) combined inversion of both arrays with Robust inversion technique