Interactive Comment on: "Modelling debris transport within glaciers by advection in a full-Stokes ice flow model" by A. Wirbel et al.

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We would like to thank Anonymous Referee #2 for detailed and helpful comments on our manuscript.

1 General Comments

Comment: *It would be a significant change, and I suggest this as being purely optional, but you may consider moving more of the benchmark results (and the detailed discussion thereof into the supplementary documents). This would serve to focus the paper more on the glaciological applications of the model. I have also made some suggestions below on where you can add more references to relevant field work that shows these types of englacial debris features. Including these may also help broaden this paper and bring it back to a more general glaciology audience.*

Response: We decided not to change the overall structure of the manuscript, as it would take away the focus from the extensive benchmark testing, which we think is an important contribution of this manuscript. Still, we included additional field work references, as suggested in the Specific Comments by Anonymous Referee #2.

Comment: *You go to great lengths to model the change in the concentration of the debris deposit as it is advected. However, all that can be seen from your figures of the 2-D test are the modelled changes in the geometry of the debris deposit. This is too bad, because, (as you say in P3, L7), the basic location and hence changing geometry of the overall debris deposit could be modelled using simple streamlines. What you bring to the table is much more powerful, however. If possible, I suggest changing the color bar / color scheme on the panels of Figure 9 so that the change in concentration as the material is advocated can actually be seen. However, I realize this may be impossible now.*

Response: As a response to this comment, we want to revisit the meaning of changes in concentration in the model results. We present a model which simulates the evolution of the debris concentration field within a glacier. The changes in concentration distribution are currently driven by advection with the glacier flow. As we treat glaciers as incompressible fluids, the resulting glacier flow fields are divergence-free and hence the concentration of debris in a control volume that is advected and deformed by the flow should not change (from a Lagrangian perspective). For a fixed volume in space, the concentration changes, as debris is advected through this fixed volume (from a Eulerian perspective) (see manuscript P3, L23-35). By modelling the concentration distribution, the model provides the required knowledge to model debris cover formation on the glacier surface due to emergence of englacial debris bands. As a result of the incompressibility assumption, changes in concentration at the edges of the debris inclusions are purely a result of numerical diffusion and mesh resolution and can hence be taken as a key metric of model performance, which we show in the benchmark tests in Sect. 5.1.

On P3, L7 we say that the location of an individual clast (single debris particle or the centre of an "undeformable" boulder) can be calculated by simple streamline tracing. However, the exact location of emergence as well as the transient shape of a polymictic debris inclusion, which will be deformed when advected through a glacier, cannot be recovered by simple streamline calculations. Even more, this becomes exceedingly difficult when the glacier geometry is changing, and hence the velocity fields. In the Specific Comments, we address the comments on the changes in concentration and the use of the color scheme in Fig. 9 in more detail.

2 Specific Comments

Comment: *P1, L18: This is still true for debris covered glacier systems that undergo no melt (cold-based alpine glaciers in Antarctica, for example. See Kowalewski et al. (2011) or Mackay and Marchant (2016). Change ”melt ” to ”ablation ”*

Response: We changed "melt" to "ablation".

Comment: *P1, L18: I suggest that you change ”. . . and transport of rock. . . ” to ”. . .or transport of rock. . . ” Although it is unusual to find these decoupled, there could be situations, where debris supply is high and yet, due to a significant slope, the transport is so efficient that you never develop a large debris-covered ablation zone.*

Response: The sentence reads: "If debris supply and melting is sufficiently high, and transport of rock material out of the glacier system is inefficient, a debris-covered glacier can develop, where a large portion of the ablation zone is covered with a continuous layer of rock material (Kirkbride, 2011)." The "and transport of rock" implies that in any case, an inefficient transport of rock material out of the glacier system is required to be able to develop a debris-covered glacier, which is also true for the situation mentioned in the comment. Therefore, we chose to leave this sentence as it is.

Comment: *P2, L2-7: Several other authors have suggested / developed this idea as well. You may want to include some additional references. i.e. Ackert (1998); Clark et al. (1998); Monnier and Kinnard (2015); Shroder et al. (2000) and others.*

Response: We now added citations for (Ackert, 1998; Clark et al., 1998) after the sentence that reads: "The implication of this process-continuum is that glaciers can transition between rockglaciers, debris-covered glaciers and clean ice glaciers through space or time as a result of the varying ice influx."

Comment: *P2, L9: Also look at the work of Reznichenko et al. (2011)*

Response: Thanks for this, we added a citation of the work of Reznichenko et al. (2011).

Comment: *P3, L11: Other field studies have shown or inferred this as well. Look at the work of Mackay et al. (2014)*

Response: Thanks for this reference, we added a citation of the work of Mackay et al. (2014).

Comment: *P3, L18: This is a reasonable assumption for this iteration of the model. However, I hope that future models versions may be able to assign spatially heterogeneous rheological properties based on debris concentration.*

Response: From a numerical point of view, it is rather easy to add spatially and temporally heterogeneous rheological properties. An excellent example is given by Aschwanden et al. (2012) with an entropy based rate factor A. Thus if a rheological parametrization based on debris content supported by convincing field data is published in the future, it can be incorporated in the model framework. However, so far the knowledge on rheological properties of debris-laden ice is limited to a small number of shear tests (e.g. Fitzsimons et al., 2001) and a recent review of available observations and model approaches (Moore, 2014) highlights both the complexity of the rheology of ice-debris mixtures and the need for further laboratory or field testing of models.

Comment: *P5, L23. P6, L1: It is not required, but I suggest you consider moving these sections (3.1 and 3.2) from the main text and putting this information into the supplemental documents. You are mostly describing tools and models that are already published. Unless you have modified them, then you don't really need to describe them here again.*

Response: We thank Anonymous Referee #2 for this suggestion but chose to keep these sections in the manuscript to provide introductory information on the tools that are used to develop the presented model.

Comment: *P6, L33. This adaptive mesh refinement is excellent. Do you also coarsen the mesh behind (upstream) of the deposit once it has transitioned down the streamline?*

Response: Due to the way the mesh refinement is implemented, no mesh coarsening is needed upstream of the advected debris features once they have been transported further downstream. In every refinement time step, the refinement is performed on a mesh that is coarse over the entire domain and only the regions of interest i.e. where debris is present and its surroundings (whose extent depends on the chosen refinement time step and the actual velocity) are refined. As the same domain-wide coarse mesh is used for every refinement time step, the regions further upstream of the actual location of the debris feature, where it has been transported through previously, do not become refined in the subsequent refinement time step any more. This spares the need for mesh coarsening. This is expressed in the manuscript at the end of P6, Sect. 3.3.

Comment: *P7, L11. You did not show results of this "comparison " in this manuscript correct? I think that you mean that in general, the results are similar to those in de Frutos et al. (2014). - which is fine - but the way this sentence is written it sounds like you have actually done the comparison and included them in your results here. For clarity, I suggest that you change the sentence slightly to read: "Comparing Results of the benchmark test in Sec 5.1 derived with our approach compare well with adaptive mesh refinement based on a posteriori error estimation (de Frutos et al., 2014), and demonstrate that ...".*

Response: This is correct, thanks. We changed the sentence to: "The results of the benchmark test in Sec. 5.1 derived with our approach compare well with that derived using adaptive mesh refinement based on *a posteriori* error estimation (de Frutos et al., 2014), this demonstrates that ...".

Comment: *P9, L21: Define uh, ue and L2 in the equation*

Response: We inserted the definitions by changing the relevant sentence to: "In order to evaluate the chosen cell area threshold, we perform convergence tests where (a) $\|c_\mathrm{h} - c_\mathrm{e}\|_{\mathrm{L}_2} = \sqrt{\sum_{k=1}^{n} (c_{\mathrm{h_k}} - c_{\mathrm{e_k}})^2}$ the L2 norm of the error between the computed finite element solution c_h and the exact solution c_e, where n is the number of computation locations, and (b) the Root Mean Square (RMS) error between the computed finite element solution and the exact solution for different cell size thresholds are computed. Therefore, we first compute the exact solution on the same mesh that is used in the finite element solution."

Comment: *P9, L26: here you have defined T as total time. I do not think that that is what you are using for T in equations (1a) and P4, L18.*

Response: In P4, L18 T is used to describe the transpose of the respective quantity. For clarity we altered it to $\top$. We also adjusted the text to use t_total instead of T to describe the total time.

Comment: *P10, L2: It is interesting that you choose a value for A that is best suited to temperate glaciers, but then use a no-slip boundary condition at the glacier-bed interface. I understand the no-slip boundary for these tests, but this condition is more consistent with cold-based polar glaciers which would have colder ice and a different value of A.*

Response: For these tests, the choice of the exact numerical value for A is not critical, as we were not trying to reproduce the actual debris transport of Haute Glacier d'Arolla, but to show the general features of debris transport within mountain glaciers. The choice of a smaller constant value of A, representative for cold-based glaciers and consistent with a "real world" no-slip boundary condition, would only cause a reduction of glacier flow velocities but not change the actual velocity patterns (see Fig. 1). This would increase the amount of time needed to transport the debris deposits downglacier, but would not induce a change of the main features and characteristics of the transport and associated deformation of debris inclusions observed in the tests. In future applications of the model, where we are planning to simulate the evolution of specific debris-covered glaciers, we will apply site-specific basal conditions and appropriate flow law parameters representative for the thermal regime of the glacier. For clarity, we rephrased the first sentence of Sect. 4.2 to: "The purpose of these tests is to demonstrate the characteristics of debris transport within mountain glaciers, not to reproduce a particular event on a specific glacier. Hence, all velocity computations are initialized with a no-slip condition at the glacier/bedrock boundary, the flow law exponent n is set to 3 and the Glen rate factor A is set to $2.4 \times 10^{-24}\ \mathrm{s^{-1} Pa^{-3}}$, a standard value for temperate ice (Cuffey and Paterson, 2010)."

Comment: *P10, L27: Remove sentence beginning: "Thereby, analysis of. . . " This sentence is unnecessary.*

Response: This sentence aims to explain the chosen test setup, therefore we kept it in the revised manuscript but changed it to: "These debris deposits of varying size, shape and location of deposition were chosen to facilitate analysis of the interplay

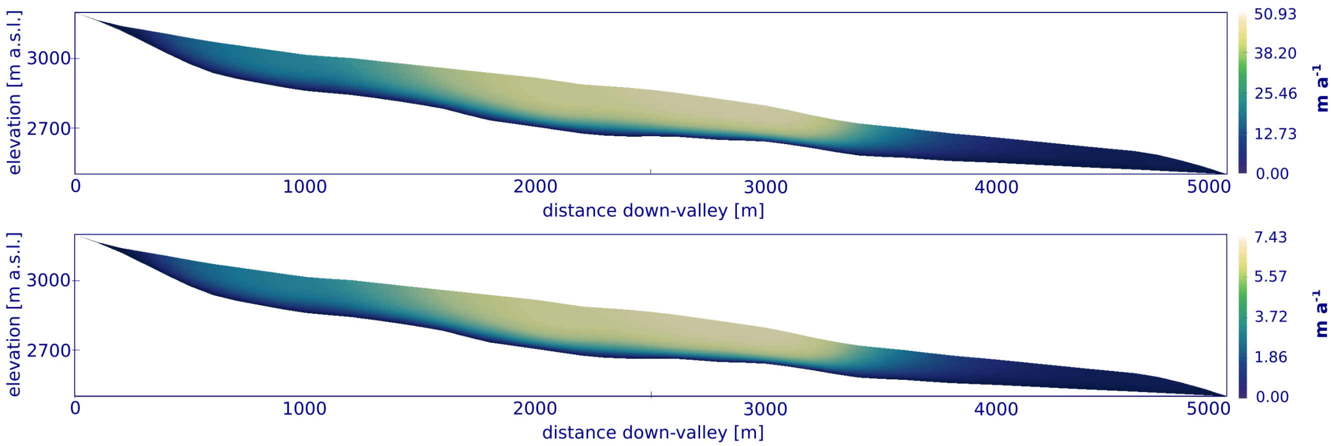

Figure 1. Velocity of 2D glacier profile. (a) temperate conditions with $A = 2.4 \times 10^{-24}\ \mathrm{s^{-1}Pa^{-3}}$, standard value for temperate ice (Cuffey and Paterson, 2010) and (b) cold-based conditions with $A = 3.5 \times 10^{-25}\ \mathrm{s^{-1}Pa^{-3}}$ a value for cold ice at roughly -10 degrees Celsius (Cuffey and Paterson, 2010)

between debris input location, deformation during transport and the zone of emergence."

Comment: *P11, L9: How big of a ”bump ” did you put in the subglacial topography? The reader has no way of knowing the characteristics of this or any other subsurface features based on the information and figures shown (see also my comment on Figure 5). Without this knowledge we cannot evaluate the impact (which I suspect is very little) that this should have on the simulation results.*

Response: We adjusted Fig. 5 to illustrate the subglacial topography of the idealized 3D glacier case.

Comment: *P13, L7: You describe the results of the rotational flow test (Fig 6) in great detail, but then barely mention the swirling flow case (Figure 7). Is there a reason for this? In any case, I suggest that you move the swirling flow case (Fig. 7) to the Sup docs.*

Response: The swirling flow test is identical to the "rotating three body problem" but using a different velocity field, which forces the features to first change their shape but then recover their initial shape at total time $t = 1.5$ s. Hence, this offers an even more challenging test case for the advection module and we included the figure to demonstrate that our model is as well capable of producing appropriate results in this test. Since the test cases are identical except the velocity field used, we keep the description short but we now changed the sentence on P13, L7 to: "Also, when the initial concentration pattern is subjected to a more complex, swirling flow (LeVeque, 1996), the results of these more challenging test simulations again show satisfactory model performance, as can be seen in Fig. 7 and Fig. A5 in the supplementary material."

Comment: *P15, L24-25: It is too bad that you did not show results from these simulations (using layer-shaped features). Although the sphere test is interesting, the layer deposits are more applicable to glaciological problems and questions. Perhaps these results were too difficult to visualize in 3D?*

Response: We are aware of the glaciological relevance of layer deposit simulations, however we chose to not present initial layer results at this stage as they deserve a detailed discussion and would be more informative if they were based on a real world example. We plan to perform such a study in the near future.

Comment: *P17, L1-3: It would be helpful if you include a reference as examples of instances where this has been the assumption.*

Response: We included a reference of the work of Naito et al. (2000), where this assumption has been made.

Comment: *P17, L1-13: It is interesting in this discussion that you have not emphasized the importance of also of now being able to quantify the changes in the debris concentration as it moves and deforms down-glacier. I would mention this. Although out of scope of this study, determining the rate of debris cover formation in the ablation zone (which is one of the main potential applications of this model once it is linked to an ablation model) is directly linked to the debris concentration and thickness of the emerging debris bands. Your model allows this to now be predicted.*

Response: The current study focuses on demonstrating a numerically stable advection scheme that can be used to predict transport and deformation of debris features, as Anonymous Referee #2 points out above. However when looking at "local" debris concentrations and their potential change over time, one has to be careful to describe the reference frame in which these changes take place. From a fluid dynamics point of view, the concentration of debris in a control volume that is advected and deformed by the flow should not change when the fluid is incompressible. This is described in the manuscript on P3, L23-35. We currently focus on that property of the model as it demonstrates the performance of our numerical implementation when we assume D, the debris diffusivity, to be very small, or even zero, and thus advection being the dominant transport process. In contrast to this Lagrangian viewpoint, a more applied (even Eulerian) viewpoint would be to investigate a fixed volume in space and time. In this case, debris concentration changes over time and such changes are very interesting for the interaction of debris with mass balance processes or the formation of emerging debris bands, as Anonymous Referee #2 points out above. We have added a sentence to the manuscript to highlight this potential application in future studies: "The model presented here allows us to simulate the advection of debris concentration through a glacier in great detail and therefore any resulting local concentration changes (Eulerian perspective), e.g. the deformation of debris deposit shape (Lagrangian perspective, cf. Fig. 10)."

Comment: *P17, L7-12: I'd would also recommend that you take a look at the work of Mackay and Marchant (2017) where englacial debris layers are directly linked to modelled changes in the environmental conditions in the accumulation zone (at orbitally-paced time scales). Mentioning that being able to test theories like this and similar shows another area in which your model can be very useful and adding this into the discussion would broaden the perceived applicability of your work.*

Response: Thanks for this reference. We included the following sentence at the end of Sect. 6.1: "This model also offers the possibility to test the findings of studies that use patterns of englacial debris distribution on Antarctic debris-covered glaciers to infer climate information at orbitally-paced time scales (Mackay and Marchant, 2017)."

Comment: *P18, L1-2: How hard would it be to implement a debris concentration - dependent rheology into your modelling framework? This would be excellent to have in future iterations of the model. See similar comment above.*
Response: In principle, the implementation would be rather simple. Compare comment above.

3 Technical Corrections

We thank Anonymous Referee #2 for the detailed Technical Corrections suggested for the text of the manuscript.

Comment: *P1, L4: Change "As debris is. . . " to "Because debris is. . . "*
Response: Done.

Comment: *P1, L6: Change ". . .surface requires that the englacial transport pathways and deformation can be known. " to ". . .surface requires knowledge of the englacial transport pathways and deformation. "*
Response: Done.

Comment: *P2. L13: Change ". . .get the full. . . " to ". . .model the full. . . "*
Response: Done.

Comment: *P2. L17: Add comma ". . . Anderson, 2016), but as. . . "*
Response: Done.

Comment: *P3. L25: Remove: " as incompressibility enforces conservation of ice density and hence ice volume. " this clause is not necessary.*
Response: We now rewrite the sentence as follows: "Assuming that ice is an incompressible fluid, and consequently that the ice flow fields must be divergence-free, any deformational patterns inducing horizontal elongation, must, at the same time, cause vertical compression. In the context of englacial debris transport, this implies that the initial debris concentration is constant for an initial control volume of ice being tracked (i.e. seen from Lagrangian perspective)."

Comment: *P3, L29: Start a new paragraph at the sentence "To solve the . . . "*

Response: We started a new paragraph at P3, L23 instead of L29 as this is one block of information.

Comment: *P3, L35: Consider deleting or moving the rest of this paragraph starting with the sentence: "In a later stage. . . " This information is better suited to the "Conclusions and outlook " section at the very end of the paper.*

Response: We included this sentence in the Introduction to clarify the scope of this paper, but also the context in which we are developing this model. We changed the sentence to: "The model presented here forms part of an envisaged fully-integrated model framework that, by including (1) a free-surface evolution scheme including debris-aware mass balance subroutines and (2) a transport model for debris at the glacier surface interacting with the mass balance subroutines, will be capable of simulating the transient response of debris-covered glaciers, with predetermined debris inputs, to a changing climate."

Comment: *P4, L6, L7: Be consistent with using either "Section " or "Sec. "*

Response: We replaced "Section" and "Sec." with "Sect." following the guidelines for manuscript preparation of The Cryosphere.

Comment: *P4, eqn. (1a) and P4, L18: define "T " somewhere*

Response: We have changed "T" to the symbol "$\top$" as a more appropriate symbol to express that a quantity has been transposed.

Comment: *P5, eqn. (5b): define $\partial\Omega_{\mathrm{D}}$. I assume that this is supposed to be $\partial\Omega_{\mathrm{bed}}$. If not, then unless D is for the diffusion coefficient, choose another notation.*

Response: We changed $\partial\Omega_{\mathrm{D}}$ to $\partial\Omega_0$ as in this case it is used to describe the boundary of the entire domain, except where input locations are prescribed. We also included the following sentence in Sect. 4.2 to describe what boundary conditions are used in the glacier tests in this study: "In the presented glacier simulations, all debris inclusions have been deposited in a single event, hence they are all initialized as inclusions within the glacier, i.e. the entire glacier/atmosphere boundary belongs to Ω_0."

Comment: *P6, L19: At sentence: "For 2D simulations. . . " Start new paragraph?*

Comment: *P6, L26: At sentence: "For 3D simulations. . . " Start new paragraph?*

Response: We start a new paragraph at the sentence: "For 2D simulations..", but as some of the concluding sentences correspond to both, the 2D and 3D implementations of mesh refinement, we keep the detailed descriptions of the 2D and 3D case in one paragraph.

Comment: *P6, L27: You have not yet introduced the refinement time step and thus this is confusing. I suggest ending the sentence with a reference to section 3.4. I.e.: ". . .at every refinement time step (see Sec. 3.4). "*

Response: We added the reference to Sect. 3.5 (as we switched Sect. 3.4 and 3.5) at P6, L18 where we first introduce the refinement time step.

Comment: *P7, L2: This sentence is awkward and should be reworded.*

Response: We changed the sentence to: "Adaptive mesh refinement strategies often employ *a posteriori* error estimation (e.g. John, 2000). The PDE is solved and the assigned error estimators and indicators are used to mark the cells for refinement and potentially coarsening. Subsequently, the marked cells become modified and the PDE is solved on the newly refined mesh. This process is repeated until the error estimators and indicators fall below a user-defined tolerance within every cell."

Comment: *P7, L15: Switch the order of sections 3.4 and 3.5. This improves readability and otherwise the reader does not know what SUPG is when you introduce it in P8, L3.*
Response: Done.

Comment: *P9, L19: Reword the sentence starting with "Here, we. . . " It does not make sense as written.*
Response: We changed the sentence to: "Here, we present results of computations using two different refinement time steps, (a) small refinement time step of $0.01\,\pi$ s and (b) a larger refinement time step of $0.1\,\pi$ s."

Comment: *P12, L9: Edit this sentence for better clarity. I suggest: "The results of benchmark tests 1 and 2 following the Bochev et al. (2004) are shown in . . . "*
Response: We wanted to use the same naming convention as in the original paper (Example 1 and 2), so we now changed the sentence to read: "Our results of reproducing Examples 1 and 2 in the numerical results of (Bochev et al., 2004)...".

Comment: *P12, L13: delete ", exemplary "*
Response: Done.

Comment: *P12, L16: You refer to the case of both refinement time steps. However, so far you have only talked about the single time step ($0.1\,\pi$) that is used in Figure 6. I know that you are also talking about the $0.01\,\pi$ time step (shown in the sup docs), but it is not clear the way it is written right now. Please edit for clarity.*
Response: We changed the sentence to: "The shapes of the concentration features are well recovered in the case of both refinement time steps (see Fig. 6b for refinement time step $0.1\,\pi$ s and Fig. A4b in the supplementary material for refinement time step $0.01\,\pi$ s)."

Comment: *P15, L22: change ". . .glacier is becoming narrower. . . " to ". . .glacier becomes narrower. . . "*
Response: Done.

Comment: *P16, L1-3: This sentence is too long and confusing and needs to be edited. I suggest deleting the unnecessary extra qualifiers: ". . .that uniformly cover wider portions of the accumulation area. . . " and "resulting in thick debris deposit but limited in area. . . "*
Response: We edited the sentence, it now reads: "In these simulations, ash fall or avalanche events that uniformly cover wider

portions of the accumulation area are included as layer-shaped debris deposits at the glacier surface. Rockfall events that result in a locally thick debris deposit are represented by a circular inclusion, as a possible remnant thereof. Both distinctly different debris inputs become severely elongated and band-like shaped during transport."

Comment: *P16, L2: Change ". . .inclusion as a possible. . . " to ". . .inclusion representative of a possible. . . "*
Response: This sentence has been changed, compare previous comment.

Comment: *P18, L15: change start of line to . . . "that is as course as possible. . .*
Response: Done.

Comment: *P18, L27: Start a new paragraph at this sentence.*
Response: Done.

4 Figure Comments

We thank Anonymous Referee #2 for helpful comments to increase the comprehensibility of the figures. If there was more than one comment concerning the same figure, we labelled the referee's comments to be able to address them point by point.

Comment: *Figure 1: Include a north arrow and scale bar*
Response: Done.

Comment: *Figure 3: - Please mark the ELA / beginning of the ablation zone. Although the reader can infer this from where the vertical velocity component passes zero from negative to positive, the addition of a simple arrow or line would be appreciated and aid in interpretation of Figure 9.*
Response: As these glacier cases are based on fixed velocity fields (based on Eq. 1 in the manuscript and given glacier geometries) and are not connected to a mass balance routine, inclusion of an ELA marker could be misleading. We added a contour line of zero vertical velocity.

After contemplating the term "steady-state" as it has been used in the manuscript and the comment of Anonymous Referee #2 above, we came to the conclusion that the term is misleading in the absence of a mass balance model in our current study. Thus the term has been replaced with "fixed velocity field" in the manuscript.

Comment: *Figures 3 and 4: - Since this geometry represents a glacier, but I suggest that you label x-axis and y-axis accordingly. i.e, "distance (m) " and "elevation (msl)*

Response: We labelled the x-axis with "distance down-valley [m]" and the y-axis with elevation [m a.s.l.].

Comment: *Figure 5:*

1. I'm not sure why you have rotated the view individual panels unless you are trying to show the overall geometry. If this is the case, then it is not effective but rather just makes the figure look rather messy and less clear. It is not required, but if possible, I suggest that you show all output in the same orientation.

2. Rather than using the axis labels x-axis, y-axis z-axis, consider using the physical interpretations for the labels (i.e. elevation, distance down-valley, distance cross-valley)

3. A wireframe showing just the glacial bed would be appreciated. As it is now, the reader has no way of knowing what the subglacial topography looks like or where the bedrock "hump " is located.

4. This is out of main scope of the debris transport focus of the paper, but I am curious as to why there is such a pronounced positive vertical velocity component at the upper right side of the glacier near the valley bend. There is compressive strain encouraging the emergence of ice here, but the magnitude surprises me.

Response: 1. We now show all output in the same orientation and for clarity added two further panels. Panel (d) shows a 2D down-valley projection of the central flowline and panel (e) shows the underlying bed geometry.

2. Done.

3. We added this as panel (e) .

4. This high compressive strain region which creates rather large but not unrealistic vertical velocities is a result of solving Eq. 1 (manuscript) for our assumed glacier geometry.

Comment: *Figure 6: - The figure is fine. The caption could use some style adjustments for readability.*

1. In this and all figure captions where you have separate panels, you do not need to use the phrase: "In (a) etc. . ...is shown " Just start with the intended panel letter and say what it is as a separate sentence.

2. Remove the sentence: "The data ranges from -0.14 *to* 1.11*. " We can see that from the figures.*

3. Move the sentence "Color scales show concentration values " to the beginning or end of the caption.

4. Shorten the final sentence to: (f) Results of the convergence test as a function of mesh refinement parameter cvol.

Response: 1. Done.

2. Done.

3. Done.

4. Done.

Comment: *Figure 7 caption:*

- Same style change comments as for figure 6.

Response: Done.

Comment: *Figure 8:*

- The first two panels fail to convey the intended information. I think that there are two separate surfaces represented for each contour (except 90) when I zoom way in, but it is barely possible to resolve these as separate at the print level of figure zoom. Also, which surface is the FEM solution and which is the analytical? These are the same color and are not labeled or marked. I recognize that they are basically the same – which may be the point of the figure, but as it is now, it is just unclear. I suggest removing the panels (a) and (b) and just leaving in panel (c) which does convey usable information.

Response: In this figure, we intended to provide a visual comparison between the FEM and the analytical solution as an additional means of illustrating the model performance. We now adjusted the figure so that the contour lines of the FEM solution and the analytical solution can better be distinguished from each other, by showing the analytical solution in solid grey and only the computed FEM solution in color and adjusted the figure caption accordingly, to highlight our intent with panels (a) and (b).

Comment: *Figure 9:*

1. The color scale (red gradient) for concentration is not effective at conveying any information as all the lines basically look like the same shade of red. It may be impossible now, but if possible, consider changing this to a multiple-color colors scheme so that the change in concentration can actually be seen.

2. Panels (a-c): Label the x-axis ('distance down-valley (m)). You probably only need to do this once for all panels.

3. Label (draw an arrow or something) the various debris layers (D1, D2, D3, etc.). Right now, it is not possible to track which is which layer.

4. Why not label the actual panel somewhere with their simulation times (24 yrs, 62 yrs, 85 yrs)

5. Panel (d): label the x and y-axis correctly and put in at least two number values on each of the axes. Otherwise, the reader has no idea of the spatial scale.

6. Why not label the actual panels with their simulation times (0.2 yrs, 8, yrs, 20 yrs, 26 yrs)

7. Caption: Remove unnecessary sentence: "Concentrations are displayed in the range of 0 to 100. " You already show this on the color scale.

Response: 1. The concentration changes at the edges of the features are based on numerical diffusion only, which is well controlled, as demonstrated by our benchmark experiments and can be nicely seen in the cross-sections of Fig. 8c. Thus changing to a multiple-colors color scheme only creates very colorful edges but does not convey the results better and we consider it as visually less appealing (Fig. 2). As concentration is given as a continuous function and we do not want to introduce subjective categories, we want to keep a sequential color scheme instead of a qualitative color scheme.

2. Done.

3. We labelled all the present debris inclusions.

4. Done.

5. Done.

6. Done.

7. As we do not want to misleadingly claim that our results have no numerical oscillations we wish to retain this sentence, but now rephrase it as: "Numerical oscillations as excursions beyond the initial values of 0 or 100 are of magnitude less than ±17 and are truncated to the data limits".

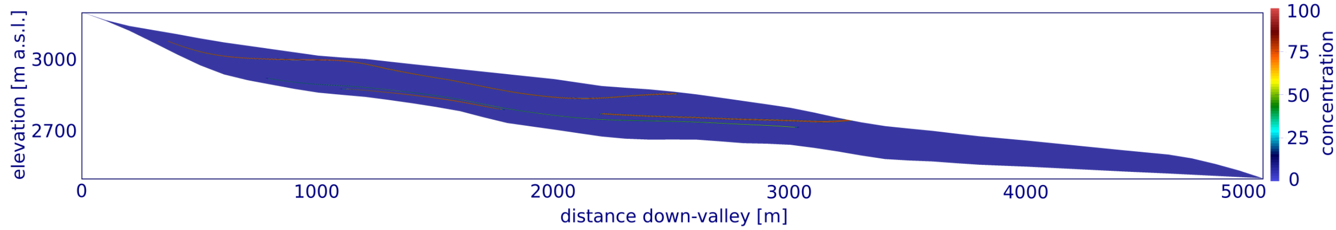

Figure 2. Results of the debris transport simulations for the 2D longprofile of Haute Glacier d'Arolla for a multiple-colors color scheme (cf. Fig.9 in the manuscript). Debris concentration at 62 years after start of the simulations is shown. Concentrations are displayed in the range of 0 to 100, numerical oscillations as excursions beyond the initial values of 0 or 100 are of magnitude less than ±17 and are truncated to the data limits.

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Comment: *Figure 10:*

1. A rough 2-d outline of the glacier (surface, bedrock along the centerline in the x-z plane and the glacier sides in the x-y plane) would be helpful for interpretation. This does not have to be exact. Please label the axis relative to the glacier model (i.e. elevation, distance down-valley, distance cross-valley)

2. Label the debris distributions with their simulation times directly on the figure. There is plenty of room and this would make interpretation easier.

3. Caption: Remove unnecessary sentence: "Concentrations are displayed in the range of 0 to 100. " You already show this on the color scale.

Response: 1. We labelled the axes as suggested and added the glacier body in a transparent style in order to help to visually locate the debris inclusions.

2. Done.

3. As we do not want to misleadingly claim that our results have no numerical oscillations we wish to retain this sentence, but now rephrase it as: "Numerical oscillations as excursions beyond the initial values of 0 or 100 are of magnitude less than ±4 and are truncated to the data limits".

5 Supplementary Material Comments

Comment: *This may be a problem with my video player (I tried several) or my download, but I cannot play some of the .avi movie files. Please check that these are not damaged.*
These play successfully:
3d_benchmark1.avi
2d_glaciercase.avi
These do not load (error):
2d_benchmark_exp1t20.avi
2d_benchmark_exp1t200.avi
2d_benchmark_exp2t15.avi
2d_benchmark_exp2t150.avi
Response: We could successfully play all the downloaded videos with the following players on different operating systems:
Ubuntu: VLC media player
Windows 7: VLC media player, Quicktime, Windows media player
macOS Sierra: VLC media player, Quicktime

Comment: *Figure A1:*
1. Please put a scale or tick marks on the x-axis and y-axis. the reader ha no idea what the scale is. Or is this dimensionless?
2. Label the color bar in panels (a) and (b).
3. In panel (b), if all the velocities are the same, then state that in the caption. Putting the "1.22 " beneath the panel looks strange and does not convey any useful information.
Response: 1. We added in the text as well as the figure caption that the computations are performed and the data in the plots is shown for the unit square ($1\ \mathrm{m} \times 1\ \mathrm{m}$).
2. Done.
3. Done.

Comment: *Figure A2 and A3:*
1. Label the panels in the left hand column (2D concentrations) with the dt used in that row. The left hand panels need x-axis and y-axis labels/tick marks (0 – 1?), otherwise the reader has no way of knowing where the profiles in the middle and right hand panels are taken from. You may also want to put two dashed lines across the concentration panels that show the location of the profiles.
2. It is odd that you put your References section in the middle of the document before Figure A2. Maybe this is something that happened in the auto-collate process during submission? I suggest just moving all Sup Doc references to the end of the document.

Response: 1. We labelled the axes and included lines to indicate the location of the right hand side profiles in the concentration plots. In order to prevent the figures from being crowded, we skipped the dt as a label.

2. Done.

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