Author's response to J. Thomas Farrar's Comments on 'On the resolutions of ocean altimetry maps'

First of all, we would like to thank Tom Farrar for taking time to read our manuscript and for his detailed and inspiring review.

We attach below the entire review made on April 2, 2019. The main concern of the review is the fact the coherence-based measure is not well adapted for estimating the resolution. The review shows several 1D general examples of the behavior of various proposed measures of the resolution (e.g., magnitude squared coherence, filter transfer function), considering low, high noise level and noise free in the input signal. For the 1D example without noise, the squared coherence and filter transfer function highlight different behavior. Similar conclusions are found for the 1D example with low noise level. In a more realistic example with high noise level, the squared coherence gives information about the input SNR but nothing about the filtering.

We agree that the filtering method for similar cutoff wavenumber can impact the coherence amplitude. This impact only happens below the cutoff frequency (Figure 1 in the review) where we should not have any coherence with independent altimetry profiles. If we have some, this means that the DUACS filter has been too aggressive and we may afford shorter correlation scales. In that case, the coherence could be misleading. To avoid that, we can consider evaluating the ratio PSD(SSH_altrack - SSH_map)/PSD(SSH_altrack), i.e., noise-to-signal ratio.

It is important to mention that our computation of the Magnitude Squared Coherence takes only into account the phase consistency between the SSH_altrack signal and the SSH_map signal (not the amplitude). For example, if the correlation scales in DUACS were larger (resulting in smoother SSH, attenuated amplitude), we could have similar phase scores but a poor PSD ratio score. We made the test, artificially smoothing the maps: we obtain the same coherence score but a low PSD ratio score. This convinced us that the coherence is indeed not sufficient as pointed out in the review, but the PSD(SSH_altrack - SSH_map)/PSD(SSH_altrack) score should well characterize the skills, by penalizing both amplitude and phase.

Hence, we propose to switch to the definition of the resolution limit of the maps as **PSD(mapping_error)/PSD(SSH_true) = 0.5** (i.e., the wavelength where the mapping error are two times smaller than the true SSH signal).

In order to illustrate the assessment of the resolution based this new definition, we performed analysis on Observing System Simulation Experiment (OSSE). The details of the simulation and methods are presented in notebooks below; and are freely available / interactively repeatable here: https://mybinder.org/v2/gh/mballaro/notebook.git/master (under the analysis_OSSE_NATL60 folder).

We have performed 3 study cases:

- STUDY CASE #1: Comparison with independent along-track (similar to the approach used in the manuscript)

- STUDY CASE #2: Comparison with NON-independent along-track

- STUDY CASE #3: Comparison with independent along-track with higher instrumental noise that mapping error

The advantage of using OSSE is that we have access to all quantity we want:

- PSD(SSH_map)

- PSD(SSH_true)

- PSD(mapping_error)

- PSD(instrumental_error)

- PSD(SSH_obs) = PSD(SSH_true + instrumental_error)

With real DUACS dataset we can only get:

- PSD(SSH_map)

- PSD(SSH_obs)

- PSD(instrumental_error)

We show in the notebooks that we can approximate the ratio PSD(mapping_error)/PSD(SSH_true) with the three quantities (PSD(SSH_map), PSD(noise_SSH), and PSD(instrumental_error)).

PSD(mapping_error)/PSD(SSH_true) = [PSD(SSH_obs - SSH_map) - PSD(instrumental_error)] / [PSD(SSH_obs) - PSD(instrumental_error)]

The main conclusions from the STUDY CASE #1 notebook are:

- the resolution estimated with the magnitude squared coherence is in good agreement with the PSD(mapping_error)/PSD(SSH_true) = 0.5 approach, linked to the fact that the signal amplitude is globally optimal at the wavelength where the phase becomes incoherent, and thus the major concern for the DUACS system is in the phase consistency between SSH_altrack and SSH_map signal rather than in their amplitude.

- the ratio **PSD(SSH_obs - SSH_map)/PSD(SSH_obs) (blue curve)** and **[PSD(SSH_obs - SSH_map) - PSD(instrumental_error)] / [PSD(SSH_obs) - PSD(instrumental_error)]** (yellow curve) are similar to the **PSD(mapping_error)/PSD(SSH_true) (red curve)** for wavelength > 70km

- the ratio **PSD(SSH_obs - SSH_map)/PSD(SSH_obs) (blue curve) or [PSD(SSH_obs - SSH_map) - PSD(instrumental_error)] / [PSD(SSH_obs) - PSD(instrumental_error)]** (yellow curve) can thus be used to estimate map resolution

- the signals for wavelength < 50 km (grey area) are not meaningful since we are under the grid spacing of the DUACS gridded product as well as in the instrumental noise level

- the sensitivity of the ratio PSD(SSH_obs - SSH_map)/PSD(SSH_obs) to the PSD(instrumental_noise) is weak for wavelength > 70km

Conclusions in STUDY CASE #1 are also valid for STUDY CASE #2.

For STUDY CASE #3 with unrealistic high instrumental noise, the ratio PSD(SSH_obs - SSH_map)/PSD(SSH_obs) becomes extremely sensitive to the instrumental noise level.

Note that this STUDY CASE #3 is not happening in DUACS processing.

In conclusion, we have made the choice to change our definition of the measure of the resolution in the revised version of the manuscript. It was previously based on the Magnitude squared coherence. It is now based on the Noise-to-Signal ratio, which is more robust penalizing both amplitude and phase consistency between the two signals. We illustrate using OSSE that the definition of the resolution based on Noise-to-Signal ratio or the Magnitude squared coherence are equivalent for the DUACS system. The main outcomes of the paper are thus unchanged. We obtain similar spatial and temporal resolutions since the signal amplitude in DUACS is globally optimal at the wavelength where the phase becomes incoherent.

We have thus performed the analysis of the DUACS maps using the NSR ratio measure. The figures have been updated in the revised version of the manuscript. A comparison of various approach to estimate the resolution capability is added and provided in the Appendix A of the paper. It includes spectral magnitude ratio approach and the transfer function approach.