Precipitation extremes on multiple time scales - Bartlett-Lewis Rectangular Pulse Model and Intensity-Duration-Frequency curves

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Reply to Comments from Reviewer #1

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We are very grateful to three anonymous reviewers for carefully reading and commenting thoroughly on our manuscript. We received highly valuable and constructive comments which very much helped to improve our work and led to new insights. We additionally got plenty ideas for further investigations.

In the following, we go point by point through all the comments and reply to them. Reviewers' comments are all repeated in this document, typeset in black. They are individually addressed, typeset in blue. Changes to the original manuscript as resulting from the reviewers comments are repeated here to ease the comparison with the original version; they are typeset in *blue italic*.

Due to some comments from the reviewers, we decided to exchange the abbreviation *BLRPM* to *OBL model* in order to distinguish the original Bartlett-Lewis model (*OBL*) from a modified version (*MBL*).

Reviewer #2

General Comments:

This paper demonstrates the use of original Bartlett-Lewis models for simulating rainfall series having precipitation extremes on multiple time scales. I believe it is an interesting paper that confirms some of the problems already indicated for the model used. More is needed in terms of discussion and a clearer extreme-value analysis, possibly involving the examination of other cell intensity distributions and proposed a new version of the model, which they called the Modified Bartlett Lewis (MBL) model. The original Bartlett Lewis model is proved efficient to explain the rainfall characteristics at all time intervals considered (1hr to 24hr) as explained by several authors such as Rodriguez-Iturbe et al. (1988) and Onof (1992), a major deficiency is its inability to reproduce the proportion of dry periods correctly. To overcome this problem, Rodriguez-Iturbe et al. (1988) proposed a new version of the model, which they called the Modified Bartlett Lewis (MBL) model. Although several studies have pointed out limitation of the original model and suggested some improvements. Onof and Wheater (1994a), for example, introduced a two-parameter gamma distribution as opposed to the original Bartlett Lewis model which considers a single parameter exponential distribution to describe the depth of a cell in order to better capture extreme events. However, the problem of underestimation of the extreme values still persists, particularly for lower aggregation levels, as described by Verhoest et al.(1997). Vandenberghe et al. (2010) found that the models demonstrated a too severe clustering of rain events.

Comments:

I would recommend the paper to be published after addressing some of the following remarks. I believe that this work could be improved by better demonstrating the advantages of the original and modified models compared to other rainfall generators (for instance, rectangular pulses models better maintain statistics at different aggregation levels), but also give an overview of drawbacks of the model. For instance, Onof and Wheater (1994) introduced a gamma distribution for the depth of a cell in order to better capture extreme events. Verhoest et al. (2010) discusses that problems still remain as infeasible cells (extremely long) sometimes occur. Vandenberghe et al. (2011) found that the models demonstrated a too severe clustering of rain events. Cameron et al. (2000) and Verhoest et al. (1997) found that these models generally underestimate the extreme values, especially for lower aggregation levels. Onof and Wheater (1993) reported problems for return periods greater than the length of the dataset. According to Cowpertwait (1998) this problem could be overcome if higher order properties would be included in the fitting procedure. Besides of being in mentioned above, the authors could validate whether the same problems occur for their simulations.

We are grateful for this comprehensive overview on the deficits associated with the original Bartlett-Lewis model (OBL) and modified versions. We used the OBL to gain an understanding of this type of stochastic precipitation models with the aim to use it in a non-stationary context in future research. Drawbacks of the OBL and also of modified versions are discussed in the literature, as mentioned by the reviewer. These deficits of the OBL might vanish (at least partially) if used in a non-stationary context where model complexity is increased as parameters are linked to large scale flow variables. This is, however, not a point to be discussed here.

Besides gaining experience for our future research plans, the manuscript we presented contributes to a) the analysis of extreme precipitation over a range of time scales in a consistent way using duration-dependent IDF curves (to our knowledge, this has been only briefly touched in Verhoest et al. [1997], and to b) the question whether the duration-dependent GEV is suitable to obtain IDF curves for these kind of models.

In the revised manuscript we introduce a paragraph reporting on the above mentioned issues in section 1:

Due to the high degree of simplifications of the precipitation process, known drawbacks of the OBL model include the inability to reproduce the proportion dry as reported by Rodriguez-Iturbe et al. [1988] and Onof [1992], and underestimation of extremes as found by, e.g. Verhoest et al. [1997] and Cameron et al. [2000], especially for shorter durations. Furthermore, problems occur for return levels with associated periods longer than the time series used for calibrating the model [Onof and Wheater, 1993]. Several extensions and improvements to the model have been made. Rodriguez-Iturbe et al. [1988] introduced the randomised parameter Bartlett-Lewis model, allowing for different types of cells. Improvements in reproducing the probability of zero rainfall and capturing extremes have been shown for this model [Velghe et al., 1994]. A gamma-distributed intensity parameter and a jitter were introduced by Onof and Wheater [1994b] for more realistic irregular cell intensities. Nevertheless, problems still remain as Verhoest et al. [2010] discussed the occurrence of infeasible (extremely long lasting) cells and a too severe clustering of rain events was found by Vandenberghe et al. [2011]. Including third-order moments in the parameter estimation showed an improvement in the Neyman-Scott models extremes [Cowpertwait, 1998].

For the Bartlett-Lewis variant Kaczmarska et al. [2014] found that a randomised parameter model shows no improvement in fit compared to the OBL model for which the skewness was included in the parameter estimation. Furthermore, an inverse dependence between rainfall intensity and cell duration showed improved performance, especially for extremes at short time scales [Kaczmarska et al., 2014]. Here, we focus on the OBL model with and without the third-order moment included. This model is still part of a well-established class of precipitation models and the reduced complexity is appealing as it allows to be used in a non-stationary context [Kaczmarska et al., 2015].

As mentioned in Section 5.2 of the manuscript, we found the OBL model to underestimate extremes merely for return levels with associated return periods much longer than our observed time series. We report this result now with a reference to Onof and Wheater [1993]. We cannot confirm a significant underestimation associated with short durations as reported by Cameron et al. [2000] and Verhoest et al. [1997], only the tendency is visible in Fig. 11, as is reported in Section 5.2. Small differences are present; we related those, however, to a problem of estimating a consistent IDF for short durations, see Sect. 5.4.

In a 1000 year simulation with the OBL, we could not find any infeasible cells as mentioned by Verhoest et al. [2010]. They discovered the problem for the modified version of the BL model. In our manuscript, we report in the discussion that in our long OBL simulation, this problem does not occur.

Motivated by this reviewer comment, we included the third moment in our objective function, following Cowpertwait [1998] and using the analytical expression derived by Wheater et al. [2006], which – as the reviewer mentions – should overcome some problems of the OBL, see Sect. 2 where we added following paragraph:

Following studies by Cowpertwait [1998] and Kaczmarska et al. [2014], we include the third moment in the parameter estimation using analytical expressions derived by Wheater et al. [2006], replacing the probability of zero rainfall in the objective function. Thus, still 13 moments are used to calibrate the OBL model. Due to comparability with other studies most of our analyses will not include the third moment though. A comparison between IDF curves of the model calibrated with the third moment and with the probability of zero rainfall will be carried out, to discuss the effect of including the third moment.

Compared to using the known problematic probability of zero rainfall [Onof and Wheater, 1994a], we could not find a systematic improvement related to extremes. This is discusses in the revised version in section 5.2. as follows:

Figure 6 shows the relative difference

$$\Delta = \frac{dd\text{-}GEV_{OBL} - dd\text{-}GEV_{obs}}{dd\text{-}GEV_{obs}} \cdot 100\% \tag{1}$$

between IDF curves (dd-GEV) derived from the OBL model $dd\text{-}GEV_{OBL}$ including the third moment in parameter estimation (red lines) or alternatively using the probability of zero rainfall to calibrate the model (blue lines), and directly from the observational time series $dd\text{-}GEV_{obs}$ for July and two quantiles: a) 0.5 and b) 0.99. Including the third moment in parameter estimation slightly improves the model extremes for July for all durations and both short and long return periods. Nevertheless, those promising results could not be found for all months (not shown) and thus we cannot conclude that including the third moment in parameter estimation improves extremes in the OBL model in contrast to findings for the Neyman-Scott variant [Cowpertwait, 1998].

Section 2.1) line 109: ...the weights, $(w_i; i = 1, 2, ..., k)$ which allow more important weight to be given to fitting some sample moments relative to others. Try to give weights given by

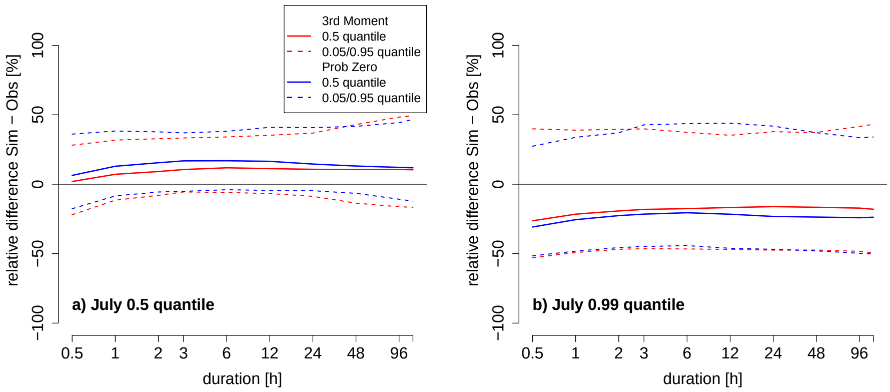

Figure 1: Relative differences between observed and simulated return levels obtained with including the third moment (red) and with using the probability of zero rainfall (blue) in parameter estimation for a) July 0.5 quantile and b) July 0.99 quantile. Dotted lines show the 0.05 and 0.95 quantile range of 1000 simulations.

$w_i = 1/Var(T_i(y))$ where $Var(T_i(y))$ represents the i^{th} diagonal elements of the covariance matrix of the summary statistics.

Vanhaute et al. [2012] investigated different objective functions specified in the following (rewritten using a notation consistent with our manuscript):

$$
\begin{aligned}
\mathrm{Z}(\boldsymbol{\theta};\mathbf{T}) &= \sum_{i=1}^{k} w_i \left[\tau_i(\boldsymbol{\theta}) \; - \; T_i\right]^2 \; (OF1) \\
\mathrm{Z}(\boldsymbol{\theta};\mathbf{T}) &= \sum_{i=1}^{k} \left\{ \left[1 - \frac{\tau_i(\boldsymbol{\theta})}{T_i}\right]^2 + \left[1 - \frac{T_i}{\tau_i(\boldsymbol{\theta})}\right]^2 \right\} \; (OF2) \\
\mathrm{Z}(\boldsymbol{\theta};\mathbf{T}) &= \sum_{i=1}^{k} 1/\operatorname{Var}[T_i] \left[\tau_i(\boldsymbol{\theta}) \; - \; T_i\right]^2 \; (OF3)
\end{aligned}
\tag{2}
$$

with the moments $\tau_i(\boldsymbol{\theta})$ derived from model parameters $\boldsymbol{\theta}$ and the empirical moments T_i estimated from the time series.

Here, we use an objective function based on OF2, using a ratio between analytic and empirical moments. In this formulation, first and second order properties are normalised by their characteristic order of magnitudes and are thus comparable. A scaling with variances as suggested by the reviewer is thus not necessary for this particular case. Additionally, we use the weights w_i from OF1 to emphasize the first moment similarly to Cowpertwait et al. [1996], see Sect. 2. We are, however, aware of objective functions like OF1 with weights being the variances of the moments as proposed by the reviewer and also by Kaczmarska et al. [2015] for the non-stationary setting; An approach we plan to pursue in the future.

Section 2 2) Give more info on the boundary constraints identified for the parameters of original model that contribute to the stability in the parameter estimates. For the original model, the values of λ that are only considered ranges from 0.01 to 0.05.

Thanks, that is definitively needed for reproducible research. We add the following to Appendix A:
Estimation of OBL model parameters follow the boundary constraints: For those parameter

Parameter	Lower boundary	Upper boundary
λ	0.004 $[h^{-1}]$	1 $[h^{-1}]$
γ	0.01 $[h^{-1}]$	10 $[h^{-1}]$
β	0.01 $[h^{-1}]$	100 $[h^{-1}]$
η	0.01 $[h^{-1}]$	100 $[h^{-1}]$
μ_x	1×10^{-9} $[mm/h]$	100 $[mm/h]$

Table 1: Boundary constrained used in OBL model parameter estimation.

ranges, numerical optimisation mostly converged into a global minimum. For the model variant using the third moment in the OF, no constraints are used.

Section 5 Results: 1. From results listed in Table 1, it is interesting to observe the higher number of storms with high cell intensity and this is contrary to our prior knowledge about less storm arrivals in dry periods like June. The occurrence of heavy rain in a short duration often induces flush floods in the city area. Form data, it is found the values of cell arrival based on the original model is smaller with high rainfall intensities, particularly for June. This implies that there is a substantial enough cell overlap which could bring extreme rainfall events. Thus, the occurrence of these realistic rainfall cells, whereas, at the hourly time scale, the annual maxima do not generally result from this model.

Thank you for pointing us to this interesting observations, we include the following in Section 5.1: *During summer months, we observe very intensive cells ($\hat{\mu}_x$ between 4mm/h and 8mm/h). However, in June and August, storm duration is relatively short ($\hat{\gamma}$ between 0.25/h and 0.35/h) which can be interpreted as short but heavy thunderstorms which are typically observed in this region in summer [Fischer et al., 2017].* This passage replaces following sentences in Section 5.1 in the original manuscript:

Large mean intensities $\hat{\mu}_x$ and short mean cell life-times $1/\hat{\eta}$ in summer correspond to precipitation being dominated by convective events. Similar, the mean cluster life-time $1/\hat{\gamma}$ decreases in summer, whereas the mean cell generation rate $\hat{\beta}$ increases.

2. Please check how the extreme events of the original model look like and compare this to the extremes of the historical series. From this you may conclude what is the problem rather than guessing that it has to do with the nature of the rainfall (maybe it is a shortcoming of the model instead! E.g. Verhoest et al. (2010))

We checked extreme events of the OBL model and visually compare them to the extremes of the historical site. We add the following figure and text to the manuscript to section 5.2. *As an example, we show segments of time series including the maximum observed/simulated rainfall in July for durations 1h, 6h and 24h as observed (RR_{obs}) and simulated (RR_{OBL}) in Fig. 7. Parts of the observed and simulated rainfall time series corresponding to the extreme events for the three different durations are shown in the left and right column, respectively. Additionally the middle column shows the simulated storms and cells generating this extreme event in the*

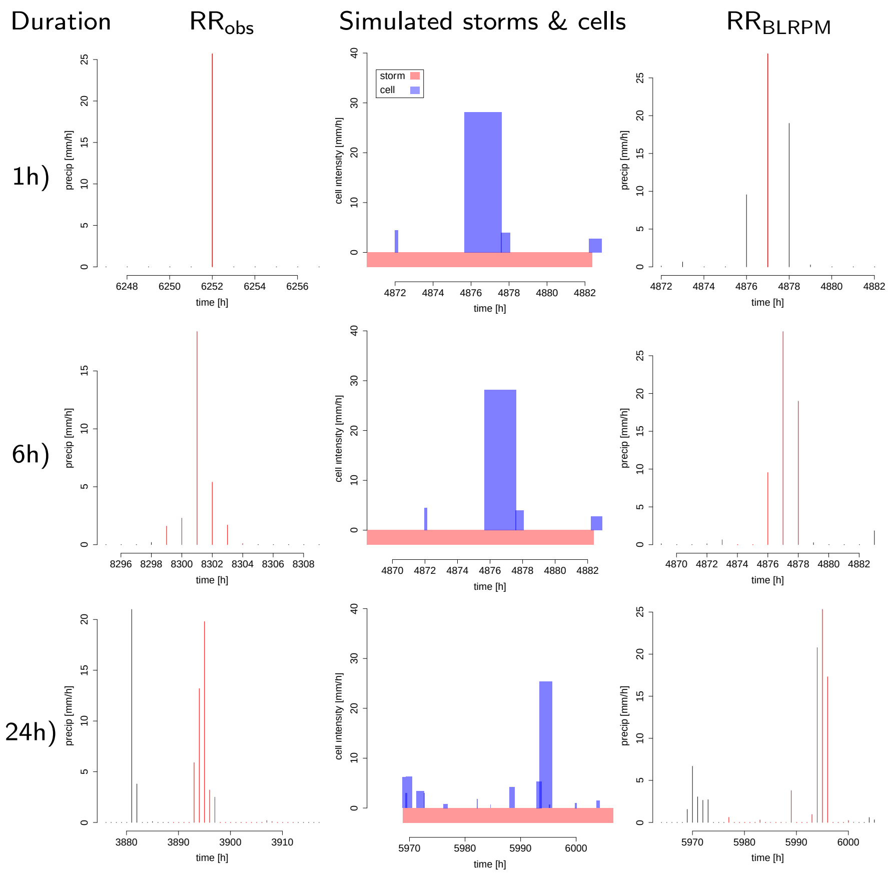

Figure 2: Visualization of July extremes as observed (RR_{obs}, left column) and simulated by the OBL model (RR_{OBL}, right column). Shown are short segments including the maximum observed/simulated rainfall (red vertical bars) at durations 1h (top row), 6h (middle row) and 24h (bottom row). Additionally, the middle column shows the simulated storms (red rectangles) and cells (blue rectangles) corresponding to the extreme event of the simulated time series.

simulated time series. Note, that we show only one simulation as an example; visual inspection of several other simulated series share the main features and are not reproduced here. For all durations, the extremes are a result of a single long-lasting cell with high intensity. In contrast to an analysis based on the random parameter BL model [Verhoest et al., 2010], these cells are neither unrealistic long nor have an unrealistic high intensity.

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