

The Cryosphere Discuss., referee comment RC1  
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## Comment on tc-2022-63

Anonymous Referee #1

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Referee comment on "Brief communication: A continuous formulation of microwave scattering from fresh snow to bubbly ice from first principles" by Ghislain Picard et al., The Cryosphere Discuss., <https://doi.org/10.5194/tc-2022-63-RC1>, 2022

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The paper applies the formulation developed by TK 21 (Torquato and Kim, Physical Review X 2021) to scattering by fresh snow. The merit of the non-local formulation is to extend the previous quasistatic model (Torquato 2002) to higher frequency so that the attenuation due to scattering is accounted for. The attenuation in scattering is included in the imaginary part of the effective permittivity. The TK21 method is supposedly valid up to  $ka = 6$  where  $k$  is the wavenumber and  $a$  is grain radius. The value  $ka=6$  means the grain radius is 1 wavelength

There are questions whether the TK21 model is applicable to snow

a. A microscopic picture of snow shows that the ice grains have irregular shapes and that they also have "stickiness" that the grains adhere together. A limitation of classical mixture formula as given in the book by Sihvola (1999) is that the mixture formula depends on the shape of the scatterers. The mixture formula is developed for simple shapes such as spheres, ellipsoids, disks etc. The problem with irregular shape is that the solutions of Maxwell equations are "discontinuous" across the boundaries between the scatterers and the background. Boundary conditions indicate that although the tangential component of electric field is continuous, but the normal component electric field is discontinuous with the normal component in air that is 3.2 times that of in ice. For well-defined shapes such as ellipsoids, such boundary value problems are solved by ellipsoid coordinates. But such problems cannot be solved analytically for irregular shapes. The TK21 model is strongly dependent on the choice of exclusion volume which is analogous to particle shape. The examples in the paper TK21 are limited to regular shapes such as spheres, disks etc. In TK21, the beta  $p_q$  in equation (33) with  $d=3$  indicates the shape of a sphere.

b. In computational electromagnetics, such boundary value problems of irregular shape/boundary have been handled by more accurate numerical techniques such as edge elements in vectorial finite element method, RWG basis function in the method of moment, and Nystrom method for volume integral equations. The popular FDTD method which is used in TK21 is not accurate. This is because FDTD uses rectangular grids in the discretization. It is unclear that the formulation of TK21 can handle the irregular shape of ice grains to correctly obey the boundary conditions on the surface of a scatterer.

c. For TK21, the methodology is based on point geometries and correlation functions associated with point geometries. In principle, the point geometry has correlation functions of infinite order which makes the solution "exact". But in practice to infinite order is not possible. Only the second order correlation function is used which means that the solutions have inaccuracies.

d. For  $ka$  extended beyond 0.5, there is incoherent field that contributes to radiative transfer equation. In addition to the attenuation due to scattering, there also is the phase matrix. This part of phase matrix has not been treated in TK21. Recently, the cross-polarization of the phase matrix at C band, X band and Ku band have drawn significant interests in microwave remote sensing of snow.

e. In Mie scattering, there are two series with two sets of coefficients: one is "electric" and the other is "magnetic". For  $ka \ll 1$ , the electric series dominate. However, when  $ka$  gets larger such as in TK21, the magnetic series contribute. It is unclear that TK21 include the magnetic series if the model is applied to dense Mie scattering

Although I have the above reservations about the applicability of TK21 to snow, I do think it is worthwhile for this paper to compute the results of attenuation of TK21 in snow and compare with other models.

I recommend the following revisions of this paper.

1, The authors should discuss the 5-bullet points a, b, c, d, and e that are raised above.

2. In figure 1, the frequency dependence power law should be extracted and tabulated. For Rayleigh scattering, it is frequency to the 4th power. The power law dependence makes the model comparisons easier to digest and remember.

3. The Mie-DMRT in figure 1 may not be correct. This is because when  $ka$  exceeds 1, more terms in both the electric series and the magnetic series should be included. It is unlikely that Mie-DMRT go off like that as in figure 1. The Mie-DMRT has weaker frequency dependence than the power law of 4. I suggest that the author delete the Mie-DMRT unless their results are correct.

4. The results in figure 2 should be done for larger grain size. At least, a new figure with  $a=0.4$  mm should be added at 19GHz. The use  $a=0.2$ mm is too small. The scattering coefficient is only a fraction per meter which is too small for real life problem. The measured volume radar backscattering at 19 GHz is much larger for a snow depth of 1 meter.

5. In equation (1), the summation over index  $n$  is up to infinity. However, in practice only second order,  $n=2$ , is used. The results in this paper are based on  $A_2$ . The expression for  $A_2$  should be explicitly given so that readers can write a computer code for  $A_2$  readily