The Cryosphere Discussions

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Reply on RC1

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Author comment on "A closed-form model for layered snow slabs" by Philipp Weißgraeber and Philipp L. Rosendahl, The Cryosphere Discuss., https://doi.org/10.5194/tc-2022-140-AC3, 2023

Dear reviewer, thank you for your comments and suggestions, which help us to improve our paper. Please find our response to each concern and remark of the review below:

Remarks:

1.) Section 2 in the manuscript presents the proposed model using continuous laminated beam or plate theory to establish a kinematic model of snowpack. This model can only analyse continuous deformation instead of fracture phenomena of snowpack especially in the weak layer. It is well known that the weak layer failure is a mixed mode damage and fracture propagation, which should be a fundamental study in assessing snow avalanches. The reviewer would suggest authors reconsidering the methodology in investigating snowpack fracture using fracture mechanics approach together with laminated plate theory.

We agree with the reviewer, that dry snow-slab avalanche release and especially the weak layer failure is governed by mixed-mode fracture processes. A fundamental problem of fracture mechanics is that the formation of new crack surfaces creates a new continuum mechanics boundary-value problem. When solved with finite-element methods, this either requires solving a new boundary-value problem with new mesh or methods of representing the newly boundaries with additional degrees of freedom such as discontinuous ansatz function (XFEM), new field quantities (phase field), degradation of element stiffness instead of removal (damage mechanics and cohesive element approaches). The present model tackles this fundamental challenge by avoiding the need for discretization and numerical concepts altogether by using established concepts of structural mechanics that yield closed-form analytical expressions that, in return, allow of near-real-time solutions of the underlying boundary-value problems. This way, they enable fracture mechanical analyses by allowing for the real-time solution of many different boundary-value problems represented by different crack states or crack advancement.
We have demonstrated this type of analysis with concepts of fracture mechanics in previous works [Ros2020a, Ros2020b].

2.) Figure 1 show a schematic model of layered snowpack, in which a weak layer is modeled as an elastic foundation using two springs with stiffness Kn and Kt. But this manuscript did present how normal and shear stiffness are coupled in the weak layer. It was mentioned bending and extension coupled in snowpack

body as a layered beam. But the failure is in the weak layer, studying coupled effects in the weak layer is much more important than the snow slab above the weak layer.

The model considers weak-layer stiffnesses Kn and Kt independently. This allows for the consideration of isotropic weak layers (where Kn and Kt are calculated from Young's modulus and Poisson's ratio such that Kn and Kt are coupled through the latter) and for the consideration of anisotropic weak layers whose normal and shear stiffnesses are measured independently.

Weak-layer normal and shear deformations (not stiffnesses) are coupled in the model through their attachment to the slab. That is, weak-layer shear deformations introduce slab bending, which in turn causes weak-layer normal stresses.

3.) Figure 4 shows some different snowpack cases with initial cracks. Fig. 4b is an upslope PST. This is a major case regarding snowpack propagation saw test (PST), some researchers have done experimental work and modelling simulation. The following published papers are for authors' reference:

a) Gaume, T. Gast, J. Teran, A. van Herwijnen & C. Jiang, Dynamic anticrack propagation in snow, Nat. Commun. DOI: 10.1038/s41467-018-05181-w.
b) Gaume, A. van Herwijnen, G. Chambon, N. Wever, J. Schweizer, Snow fracture in relation to slab avalanche release: critical state for the onset of crack propagation, The Cryosphere 11 (2017) 217–228, https://doi.org/10.5194/tc-11-217-2017.
c) Jiye Chen, Blair Fyffe, Dawei Han and Shangtong Yang, Predicting mixed mode damage propagation in snowpack using the extended cohesive damage element method, Theoretical and Applied Fracture Mechanics 122 (2022) 103567, https://doi.org/10.1016/j.tafmec.2022.103567.

Reference c summaries the weak layer's fracture mechanisms in the upslope PST case. It is well accepted to recognise a mixed mode damage and fracture in the weak layer under compression and bending. This can also be seen from Fig. 4b, the left part of segment 2 is under compression and shear and the right part under tension and shear. The left part is subjected to both compression and bending in the same way in compression. The bending caused tension and compression have opposite effects on the right part. The left part of segment 2 would fail by compressive crushing and shearing. Once the weak layer fails by the mixed failure mode with crushing and shearing, the snow slab moves down due to gravity. Reference c proposed a new mixed mode damage criteria and successfully assessed the weak layer's failure initiation and propagation. But this manuscript concluded that the upslope PST is a mode I crack (opening). This is questionable. This manuscript has not yet considered the micro damage phenomenon in the weak layer because of the continuous mechanics model used, which cannot reflect the reality of failure in the weak layer. Snow is a porous material with micro crystal construction, which can be easily crushed by compression.

When we speak of mode I anticrack propagation we refer to the symmetric nature of the deformation field [Bro1999]. Because we consider anticracking, mode I corresponds to weak-layer crushing and collapse. This agrees with the reviewer's understanding of the fracture process described above. Further details of this definition are given in a previous The Cryosphere publication by the authors [Ros2020b]. We will revise the manuscript to make this more clear.

4.) Equation 19 and Figure 16 present a total energy release rate in the weak layer: G = G1 + G2. But this manuscript has not used the G for assessing crack propagation. Theoretically, total G is not total fracture energy criterion Gc in the

mixed mode case, it contains contributions from G1c and G2cin a coupled model. This important point was missed by this manuscript.

Of course we agree that mixed-mode crack nucleation and propagation is governed by the interaction law of the mode I and mode II fracture toughnesses as discussed [Ber2023]. However, we are not aware of any experimental work by other authors that provides this relation. On the contrary, the magnitude of the mode I and II fracture toughnesses in pure modes are not well defined. We are currently working on new methods to identify mixed-mode fracture envelopes of weak interfaces [Ada2022].

However, it is not the aim of the present work to establish a novel failure criterion. We have proposed a mixed-mode finite fracture mechanics approach for non-layered slabs in an earlier work [Ros2020b] and we will incorporate the present model for layered slabs in this framework in the future.

5.) On page 9, the last sentence: " Energy release rates obtained using weak-interface kinematics cannot capture very short cracks .... ". This is a case from the kinematic model. However, if energy release rate obtained by fracture mechanics in weak layer, it can certainly capture any crack length. This has been done in standard fracture benchmarks of laminated composites in literatures.

We agree with the reviewer that this comprises a limitation of the model. This is the reason why we indicate this as the limitations within the chosen modeling approach. In the present case, the chosen kinematics do not capture extremely short cracks well. However, many established failure analysis concepts that use stress, strain, strain-energy density evaluation at a distance or the concept of finite fracture mechanics, do not make use of the localized quantities so that this limitation does not necessarily render a problem.

6.) Section 3 in this manuscript presents validation. It compared the outcomes from the proposed analytical model with FEM based continuous deformation analysis regarding deformation and stresses. There is no usage of test results in validation. This is also questionable. Firstly, the modelling results from FEM based continuous analysis are not reliable when the FEM mesh with cracks, or without validation by test work, the FEM results would not be accepted to validate other new models regarding fracture issues.

The use of FEM results to compare two solution methods of the same boundary value problem of structural mechanics models is common practice [Zie2005]. When composed carefully, FE models are very well suited for boundary-value problems including cracks, notches, or other stress concentrators [Kun2013].
The FEM model used in the present work has been conducted carefully, including a mesh sensitivity analysis to rule out mesh effects. Since the focus of the present work is the development of a real-time closed-form model for layered snowpacks and since we use FEM just for validation purposes, we do not believe the work would benefit from a detailed description of the FE model. Instead, we will add a reference to a previous publication, that discusses the FE model in detail – with the only difference being the slab layering.

Moreover, we validate the displacement field produced by our model using full-field DIC displacement measurements in Fig 7. (anonymous reviewer #2 agrees).

7.) On page 12, the last paragraph, " Experimental validations are challenging since direct measurements of stresses are not possible and displacement measurements require considerable experimental effort ". This is truth. However, above references a and b reported critical initial crack length in PST experimental work, which causes crack propagation and sliding of snow slab above weak layer, and the critical initial crack length related bending moment

can be used for validation if the critical load can be provided by the analytical model in this manuscript. But this manuscript failed at the validation in this way. Abovementioned reference c has completed a similar validation.

Thank you for this suggestion. We will incorporate an estimate of the weak-layer (crushing) fracture toughness that can be identified from PST data in our revised manuscript.

8.) In example section, manuscript presents a lot of information about stress and deformation distributions in snow slab in different cases. The reviewer thinks this information is less important in assessing snowpack fracture. The outcomes from this proposed model are not only difficult for validation and also in application for assessing snowpack fracture initiation and propagation. As a closed-form solution, it is expected to provide precise failure response which shows a load-deformation curve. Thus, it can clearly present elastic stage, damage point, damage accumulation and final crack point.

The stress distributions and energy release rates of snowpacks with intact and collapsed weak layers are fundamental for the analysis of weak-layer failure [Sch2003, Sch2016]. We agree that a closed-form analytical solution providing a precise failure response of a slab-weak-layer configuration is of great interest. This is precisely what our model aims at with great efficiency. We provide it as public source-code to other researchers on github.com.

Whether brittle failure phenomena such as weak-layer fracture really show damage accumulation is worth a long discussion. For instance, does damage accumulate in the weak layer if many skiers travel over the same point? Does that mean we should avoid well-traveled slopes – which is certainly counterintuitive?

9.) This manuscript mentioned a basic concept of a failure mode in weak layer: mode I (collapse) or anticrack. Although this terminology was used by some researchers in their early work. The reviewer thinks that this terminology is not in the line of the basic concept of fracture mechanics about mode I: opening crack. The wording of collapse or anticrack would not be a right description of the failure mode in opposite way of mode I. Abovementioned reference c properly suggested a terminology of crushing damage by compression, and proposed a mixed mode damage criteria with crushing damage and shearing crack (mode II) to assess the failure in weak layer under mixed loading.

As already discussed in point 3.), we have used the usual way to consider symmetry of the displacement field to distinguish different loading modes of a crack [Bro1999]. This has been done by many researchers that studied anticracks, e.g. in snow [Hei2008, Gau2018], in compaction bands of limestone [Ste2005], pressure (dis)solution in rocks [Fle1981]. We follow the highly influential thoughts of Heierli et al. [Hei2008] in our denomination "collapse" but agree that crushing is an accurate description of the phenomenon, too.

10.) A closed-from should clearly present the failed load and residual strength when crack propagates through the failure response or load-deformation curve. The reviewer thinks that this manuscript has not reach the purpose of investigating and predicting the physical processes that lead to the formation of dry-snow slab avalanches.

We agree with the reviewer that some of these features would be of interest in a failure model. The present model is not a failure model.

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