Reply on RC1

Jonathan Kingslake et al.

Author comment on "Grain-size evolution controls the accumulation dependence of modelled firn thickness" by Jonathan Kingslake et al., The Cryosphere Discuss., https://doi.org/10.5194/tc-2022-13-AC1, 2022

We would like to express our thanks to the review for the review and the positive words about the paper. We are pleased to hear they find it timely and important.

The reviewer's comments are reproduced below and our responses are shown in **bold**.

1. General

In this paper, the authors analyze the role of snow accumulation rate on the compaction of dry firn layers on top of ice sheets. The authors' contribution is timely and important; as summarized in their introduction "Some [models] treat accumulation as a boundary condition, as it is in other ice-deformation modelling contexts. Others include b [accumulation rate] in their constitutive relations." (with the citations removed and two parenthetical additions). This is important because in Herron and Langway (1980), the gold standard for firn compaction, the empirical compaction constitutive relation is proportional to accumulation rate, meaning that if it stops snowing, compaction ceases. However, in the physical system, compaction should continue if snow accumulation stops; this is the problem that Kingslake et al. address in this paper. Building from Appendix B of Arthern et al. (2010), the authors develop a full numerical simulation of very sensible compaction equations including grain growth and complement these results with steady state ordinary differential equation (ODE) solutions to reduced models. Arthern et al. (2010) contends that if the grain size (rs2 in the parlance) is near zero at the surface, then the system approaches Herron and Langway (1980), with a direct dependence on the accumulation rate. Here Kingslake et al. show that in the rs2 □ 0 limit, the effects of porosity advection and grain-size advection cancel and the model firn depth is independent of the accumulation rate — another puzzling result of the Arthern et al. (2010) work. Kingslake summarizes "models that include viscous firn compaction and grain size evolution (e.g. Arthern et al. (2010)) are potentially capable of a much richer array of response to accumulation rate than is usually recognized." This is a very nice paper that cogently examines the formulation of the predominant firn compaction model. With some small tweaks, this paper is certainly worthy of publication in The Cryosphere.

2 Remarks

- The model observation that the firn depth z830 increases with accumulation rate is interesting and challenges my intuition. The weakest dependence comes from Arthern et al. (2010) with rs2 □ 0, where the depth is independent of b. Yet many of the places

with the deepest firn columns, e.g. domes of East Antarctica, have very low accumulation rates, leading me to think that there is a regime where z830 ☐ 1/b. Is that misguided? This would seem to imply that if the accumulation rate increased in East Antarctica, then the firn depths would also increase. This seems to be backwards from the current firn thickness / accumulation rates in Antarctica, where thinner firn thicknesses exist on the coasts where it snows more.

Yes, this model predicts that firn thickness increases with increasing accumulation rate. The primary reason for the thicker firn in interior East Antarctica versus other parts of the ice sheets is the lower surface temperatures. Firn compaction rate is a strong function of temperature, so the low surface temperatures (and hence low temperatures throughout the firn column) lead to slow firn compaction. To isolate the relationship between accumulation rate and firn thickness, we left temperature uniform and constant throughout the modeling. In observational records of firn thickness the relationship between accumulation rate and firn thickness is made less clear by the positive relationship between temperature and accumulation rate - in general, higher temperature leads to higher accumulation. So in cold places like East Antarctica the low temperatures cause slow compaction but they also cause low accumulation rate, the effect of which in this case is insufficient to counteract the effect of the low temperatures and the net result is relatively thick firn.

This is discussed very briefly in the new paragraph quoted below in lines 431-440.

- Interestingly, rather than leaving the temperature constant for the different accumulation rates, am I understanding correctly that the temperature also changes? It seems like it might be better to change one thing at a time to see the effect of that change more clearly? Or do you change them independently? More generally, I am curious about the effects of temperature on the results.

To isolate the relationship between accumulation rate and firn thickness, we assumed a uniform and constant temperature throughout. We leave exploration of temperature effects to future work.

We have added the following paragraph to the discussion (L431-440) section to discuss this issue.

"To isolate the effects of accumulation rate on firn thickness we assumed a uniform and constant temperature. However, temperature is a first-order control on firn thickness in this model, through its impact on grain growth and on firn compaction (Section 2.3). Surface temperatures vary regionally with climate. This variability would need to be taken into account in any attempt to compare model results to observations (e.g., Montgomery et al., 2018), particularly because accumulation rates generally increase with increasing temperature (e.g., Frieler et al., 2015, Dalaiden et al., 2020), complicating the simple relationship between accumulation and firn thickness predicted by our model. Temperature also varies in time, causing transient vertical variability in temperature throughout the firn column, in part, through advection of heat. Further model development and analysis will be required to assess how the modelled accumulation dependence of firn thickness differs in this scenario, in particular in the case when accumulation also varies in time. The latter, presents the possibility of complex interplay between advection of porosity, grain size, and heat. We leave exploration of this to future work."

We hope that the new paragraph, along with the mention of this isothermal

assumption in the second paragraph in section 2.1 (L80-81) make it clear to the reader that we are making this assumption.

Specific comments

Figure 2(left): the ODE and full compaction model results are indeed indistinguishable. Maybe make one dashed (narrow) and the other solid (wide) and embed them? Or use symbols for the full model?

We were unable to find a way of making the two sets of curves distinguishable: using markers did not help because there are many grid points and therefore markers merge together to make one thick line, while using two solid, very thin lines for each curve detracts from the readability of the plot, as currently the line thicknesses and styles uniquely distinguishes the curves without the need for the colors, helping color-blind readers. If we were to remove the line styles this benefit would be lost.

We decided to not change the line styles, but are happy to revisit this decision if the reviewer or the editor would like us too, and have suggestions for the best compromise of readability and clarity.

Figure 2(right): would the absolute difference normalized by the absolute maximum value of the quantity be more useful?

Normalizing this plot would scale the curves, but not change the plot significantly because the model is nondimensionalized. We chose not to make this change because the original approach seems like the simpler, and clearer, of the two options.

Figures 3, 4: it could be useful to mark the location $z830$ on the plots using a symbol, so that the trend from the inset is visually clear. Maybe it could go on the velocity side, so that it doesn't get compressed at the bottom of the porosity plot.

This is a good suggestion. We have added markers to the right hand panels and a horizontal line to the left panels, and updated the captions appropriately.

References

J. Arthern, D. G. Vaughan, A. M. Rankin, R. Mulvaney, and E. R. Thomas. In situ measurements of Antarctic snow compaction compared with predictions of models. J. Geophys. Res., 115(F3), 2010. doi: 10.1029/2009JF001306.

M. Herron and C. C. Langway. Firn densification: an empirical model. J. Glaciol., 25(93):373–385, 1980. doi: 10.1017/S0022143000015239.