

The Cryosphere Discuss., referee comment RC2  
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## Comment on tc-2021-72

Anonymous Referee #2

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Referee comment on "Elements of future snowpack modeling – part 1: A physical instability arising from the non-linear coupling of transport and phase changes" by Konstantin Schürholt et al., The Cryosphere Discuss.,  
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Review on "Elements of future snowpack modeling – part 1: A physical instability arising from the non-linear coupling of transport and phase changes" by Konstantin Schürholt, Julia Kowalski, and Henning Löwe.

The paper describes consequences of the incorporation of vapor transport in snowpack models on the numerical schemes in idealized settings. The water vapor transport in combinations with phase changes introduces additional non-linear terms in the heat and mass balance equations which the standard numerical schemes do not have to take care for. In three case studies the non-linear differential equation system is solved with a python-based Finite Element Framework for 1D snowpack models.

It is shown convincingly that currently used continuum-mechanical models derived through homogenization or mixture theory yield similar results for homogeneous snowpacks of constant density. However, if the snow density and temperature varies significantly with depth phase changes result in non-linear advection of the ice phase. This advection potentially exaggerates density variations which potentially initiates wave instability in the continuity equations. A linear stability analysis reveals that the wave instabilities are caused by the density dependence of the effective transport coefficients.

The work deserves publication. The presented analyses are sound and the reasoning behind the several steps of the analyses is clear.

However, the quality of the presentation of the results is poor. This holds for the textual form but also for the readability of the figures. The sentence construction is often unnecessary complicated making reading of the manuscript a tedious work. Below I will present some examples and suggest corrections but the list is not complete. I recommend to get help from a native English speaker.

line 4: Spell out PDE

line 5: Skip 'solely'

line 7: Skip 'different,'. Take care of coma.

line 8: "For heterogeneous situations in which the snow density varies significantly with depth, we show that phase changes in the presence of temperature gradients give rise to a non-linear advection of the ice phase that amplifies existing density variations." I suggest: "When snow density varies significantly with depth, we show that phase changes in the presence of temperature gradients give rise to non-linear advection of the ice phase amplifying existing density variations."

line 18: "As hypothesized in recent work on shallow tundra snowpacks (Barrere et al., 2017; Domine et al., 2016) persistent temperature gradients throughout the season may contribute to the depletion of snow density at the bottom of the snowpack due to persistent upward vapor fluxes." I suggest: "Persistent temperature gradients throughout the season may contribute to the depletion of snow density at the bottom of the snowpack due to upward vapor fluxes, as has been hypothesized for shallow tundra snowpacks by Barrere et al. (2017) and Domine et al. (2016)."

line 31: "Lastly (Hansen and Foslien, 2015) was revisiting the problem of coupled heat and vapor transport using mixture theory which led to a more restrictive set of transport equations that rely on the assumption that the vapor concentration is always close, but not exactly in equilibrium with temperature. While the existing vapor schemes largely differ in the form of the effective transport coefficients, there is a general agreement on the basic type and form of the partial differential equations (PDE), that govern coupled heat and diffusive vapor transport in snow." I suggest: "Hansen and Foslien (2015) revisited the problem of coupled heat and vapor transport using mixture theory leading to a more restrictive set of transport equations. They rely on the assumption that the vapor concentration is always close, but not exactly in equilibrium with temperature. While the existing vapor schemes largely differ in the form of the effective transport coefficients, there is a general agreement on the basic type and form of the partial differential equations (PDE) governing coupled heat and diffusive vapor transport in snow."

line 42: "The first attempt to solve the vapor diffusion equation in a snowpack model was recently undertaken by (Jafari et al., 2020) who equipped the model SNOWPACK with a vapor transport scheme as a non-linear reaction-diffusion equation." I suggest: "Recently Jafari et al. (2020) equipped the model SNOWPACK with a vapor transport scheme in form of a non-linear reaction-diffusion equation. It is the first attempt to solve the vapor diffusion equation in a snowpack model."

line 43: "The numerical solution requires time-steps of 1 min and mesh sizes of 1 mm to avoid "numerical oscillations" that were observed, even within an implicit, unconditionally stable numerical scheme." I suggest: "Even within an implicit, unconditionally stable numerical scheme, "numerical oscillation" requires very small time-steps of 1 min and mesh sizes of 1 mm."

line 50: "Phase change processes in seasonal and polar snowpacks are commonly of interest on long time scales ideally using coarse meshes and large time steps to meet requirements for climate modeling." I suggest: "Phase change processes on seasonal time scales in polar snowpacks are important for climate modeling ideally adequately simulated on coarse meshes with long time steps"

line 51: "It is therefore necessary ..." -> "Therefore, it is necessary ..."

line 62: "It is the aim of the present paper to advance ..." -> "We aim to advance ..."

line 73: "This is confirmed by an analytical, linear stability analysis which relates unstable behavior to the density dependence of the effective (heat and mass) diffusion constants. The results suggest that previously observed oscillations in the numerical treatment (Adams and Brown, 1990; Jafari et al., 2020) were not numerical problems but may

rather have hypoallergenic physics." I suggest: "This is confirmed by an analytical linear stability analysis attributing the unstable behavior to the density dependence of the effective heat and coefficients. The results suggest that previously obtained oscillations in the numerical schemes (Adams and Brown, 1990; Jafari et al., 2020) are physical and not numerical artifacts."

line 97: " $\rho_i$  is the density of ice density (assumed to be constant)" -> " $\rho_i$  is the constant density of ice"

line 110: "in (Calonne et al., 2014)" -> "in Calonne et al. (2014)"

line 117 to 123: I suggest to rewrite the paragraph. I could not understand it.

line 125: "the same is not true" -> rewrite

line 128: "into a single one that no longer" -> rewrite

line 136: "was not been considered" -> "is not considered"

line 140 to 143: I wonder why here the models are referenced as 'Calonne' and 'Hansen' but not in the paragraph in line 124 to 129. Homogenize it.

line 161: "In summary, all symbols and parameter values used in this study are provided in Table 1." -> "We summarize all symbols and parameter values in Table 1."

line 171: "For the spatial discretization we note that the non-linear PDE systems, of interest can be formally rewritten in the form" -> "The non-linear PDE systems of interest can be rewritten in the form"

line 177: You should explain what 'small support' means.

line 190: "The vapor equation has by far the fastest dynamics, followed by the energy.190The ice mass balance instead has a much slower dynamics." I suggest: "The vapor equation has by far the fastest dynamics, followed by the second fastest, the energy equation. The ice mass balance equations has a much slower dynamics."

line 191 to 193: Hardly understandable sentence. Rewrite.

line 200: "... which is known to be stable and converge of second order for linear operators." Do you mean "and convergent at second order"?

line 209: Add 'we': "For each of the three physical scenarios we evaluate three model formulations:"

line 214: "The first scenario is taken from (Calonne et al., 2014) who investigated the response of a homogeneous snow layer to transient heating. To this end we use the IC" I suggest: "The first scenario is proposed by Calonne et al. (2014) and investigates the response of a homogeneous snow layer on transient heating. The initial conditions are:"

line 224: "For this combination of IC and BC we obtain the results in Figure (1) where the solutions of all three cases at  $t=10h$  are shown." I suggest: "The solutions of all three cases we obtain for this combination of IC and BC are shown in Figure 1 for  $t=10h$ ."

Figure 1 and Figure2: The lines can be hardly identified. I suggest to increase the line width.

Figure 3: Same here. If the lines are too close to each other I suggest to write that in the figure caption.

line 302: "The fact that  $D_{eff}$  decreases, and  $k_{eff}$  increases with ice volume fraction, decreases the ice flux functional  $G$  in high density regions over lower density regions." This sentence is not understandable.

line 316: "This is interesting, as it suggests that these waves are true, intrinsic features of the full Calonne model equations, rather than an artifact of the numerical scheme." I suggest: "This suggests that these waves are an intrinsic features of the Calonne model equations rather than an artifact of the numerical scheme."

line 320: "To comprehend the oscillatory nature of the solution we analyzed the problem theoretically within perturbation theory." I suggest: "We use perturbation theory to comprehend the oscillatory nature of the solution."

Figure 5: Same as for the other figure. Increase in width and if lines overlay mention it in the caption.

line 335: Skip "To the end"

line 336: "small" -> "thin"

line 337: "can be always" -> "is always"

line 245 Is (28) correct?

line 386 to 388: Split the sentence in several sentence. Hardly understandable.

line 444: "As a nasty coincidence". Very sloppy language.

line 479: "the the"

line 496: "equation ... equation"?