

The Cryosphere Discuss., referee comment RC2  
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## **Comment on tc-2021-41**

Anonymous Referee #2

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Referee comment on "Controls on Greenland moulin geometry and evolution from the Moulin Shape model" by Lauren C. Andrews et al., The Cryosphere Discuss.,  
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Apologies for taking a long time to produce this review.

This is a very interesting paper that presents a new model for the evolution of moulin geometry, and explores how the results of this model for moulin shape and water level depend on various model parameters. It is argued that moulins comprise a sizeable fraction of the englacial-subglacial drainage system in Greenland, and that the time-evolution of their volume is a potentially important feature to include in englacial/subglacial models, offering improvements over a model that assumes a static moulin volume.

The study is an interesting one and I believe it deserves publishing in some form. However, I do have quite a lot of detailed questions, and some concerns, about the ingredients that go into the model. I will focus this review largely on these model details, from section 2 of the paper. Some of these may be sorted out by clarification as to what equations have actually been solved. As a general comment, there appears to be quite a lot of duplication of notation, which overcomplicates the presentation of the model and causes some confusion. I think it would also be helpful to express the physics in terms of differential equations rather than discrete increments that implicitly include time-steps.

## Major comments

Section 2.2.1 - the rationale for modelling the moulin cross-section with this strange egg shape was weak for me. It significantly complicates the model to do this, rather than to assume it has a circular cross-section, and it was not at all clear to me that there was any great advantage in doing so. It is also not clear how  $r_1$  and  $r_2$  are separately evolved, and this needs to be made clearer. I ended up with the impression that the difference is likely because the open channel flow above the water line gives rise to a change in one of these but not the other; but below the water-line it seemed that  $r_1$  and  $r_2$  would evolve identically and therefore stay the same, assuming they start the same? However, this should be made clear by telling us what exactly are the equations that govern the evolution of  $r_1$  and  $r_2$ . I would, at the same time, encourage the authors to think about simplifying things and assuming circular symmetry, since I think many of the results would still apply, and I think it would give a model that is more likely to be adopted by others.

Section 2.2.2 - I had great difficulty following the treatment of elastic deformation, and am slightly concerned that this is not dealt with correctly. In particular, a number of figures (figure 6, figure 9) compare viscous and elastic 'deformation' as a \*rate\*, with units m/d. Elastic deformation is not a rate - it is an instantaneous deformation and it results in a displacement (relative to some reference state) that is fixed, for fixed stress - in this context, that is the change in radius given by (4). Presumably this must be viewed as relative to some 'reference' radius that can evolve in time due to viscous deformation and phase change. The elastic displacement in (4) does itself evolve in time due to changes in water pressure, and therefore gives rise to a deformation rate that is  $d(\Delta r_E)/dP * dP/dt$ , i.e. proportional to the rate of change of water level, and perhaps that is what is being plotted in these figures, but I did not really have this impression. If that is indeed what is meant, note that the elastic deformation rate depends on the rate of change of  $P$ , not on  $P$  itself, so whether the pressure is above or below overburden is irrelevant to the sign of the deformation rate (it is instead a question of whether  $P$  is increasing or decreasing).

This concern is tied up with the question above of how exactly  $r_1$  and  $r_2$  are evolved. It seems to me that you would want to have 'reference' values of these that evolve according to the viscous processes; they satisfy an equation of the form  $dr/dt = \text{melt-back} - \text{viscous closure}$  (very similar to the subglacial channel in (28)); and then you want to add the elastic deformation given by (4) on top of those evolving reference values to get the actual radius at any instant in time.

In equation (4), I would be inclined to simply ignore the deviatoric stresses, which I expect are relatively small in most cases compared to the effective pressure  $P$  (it was not clear to me what you have actually assumed for them in the examples). Given that you are comparing with a null model which contains no moulin physics whatsoever, I think there is some advantage in not making this one overly complicated! Note that there are in any case some missing brackets in this equation. In equation (5), the first term is presumably set to zero for  $z$  larger than  $h_w$  (i.e. above the water line)? This equation could be made more consistent with (6), which is essentially the same thing, but where  $P$  is now called  $\sigma_z$ . (6b) should again be zero for  $z$  larger than  $h_w$ , I think.

Equation (9) is a strange way of discretising the time-derivative and this is where confusion starts to arise as to how  $r$  is actually evolved, because this gives an incremental change in  $r$  (both  $r_1$  and  $r_2$  ?) due to only viscous processes, and it is not clear how this is combined with the changes due to phase change and elastic deformation. The viscous closure of a moulin due to (7) is essentially identical to that for a subglacial channel as described in (27) and as described by Nye (1953) for the closure of a borehole. I think it would be helpful to express it as a contribution to the time-derivative  $dr/dt$ , as (effectively) done in (27).

Section 2.2.2.2 (I don't think I've ever seen quite so many subsections!) - The downstream deformation of the ice is interesting, but it wasn't clear to me how it is incorporated into the model. It seems like it translates the 'centreline' of the moulin? But doesn't affect  $r_1$  and  $r_2$ ? So does it actually have any effect on the rest of the model or is it just relevant for the visualisations like in figure 8? The formula in (10) assumes no slip at the bed, which is presumably not always going to be the case?

Section 2.2.3 - I was a bit confused why melting and refreezing are treated separately - you could simply write down an energy balance that allows for either to happen automatically, depending on the relative magnitude of turbulent heating and the conduction into the ice, without having to have any 'switch' between melt season and not. In (11), I would have thought that the  $dT/dx$  should really be a  $dT/dr$ , i.e. the radial temperature gradient away from the (roughly) cylindrical moulin; the distinction between

them is quite important because conduction around a point source in two dimensions (ie. in the  $x,y$  plane) is very different from conduction in one dimension (i.e. in  $x$  alone). That said, solving the heat equation in the ice for each different  $z$  seems a lot of work for a model of a single moulin, and I wonder if a reasonable approach would be to simply \*estimate\* the temperature gradient at the moulin wall,  $dT/dr$ , as  $(\Delta T)/r_m$ , where  $\Delta T$  is the temperature difference to the far-field ice and  $r_m$  is the moulin radius. That would be consistent with the way you incorporate the estimate of sensible heat in (18) when considering melting.

Is  $\Delta r_t$  in (19) the same as the melt rate  $m$  in (14)? And what exactly is  $Q$  here, in relation to the other  $Q$ s mentioned later ( $Q_{in}$ ,  $Q_{out}$ ,  $Q_{base}$ )? If I understand the picture correctly I think it ought to be  $Q_{out} - Q_{base}$ , since that's the flow out of the moulin into the subglacial channel. This could all be made clearer with more consistent notation. I couldn't follow what is used for the melting in the open channel zone on L287-295; it says you use (17), but that doesn't seem helpful. I would have thought you want to use something more like (19), but with  $Q$  replaced by  $Q_{in}$ , and with a modified hydraulic radius and perimeter.

Section 2.2.4 - Equation (22) needs to include  $Q_{base}$ , similarly to equation (24). In fact, there seems to be some inconsistency and duplication between (22), (24) and (29). These equations are all expressing mass conservation, and (22) and (29) are really the same equation (I assume that the  $m$  in (29) must include the freezing rate  $-\delta$  as well). But I think they should include  $Q_{base}$  if you're going to include  $Q_{base}$  in (24). And I think (24) should really include some terms to account for the rate of change of the cross-sectional area (it comes from inserting  $V_m$  as the integral of  $A_m$  from 0 to  $h$  in (29)).

Section 2.2.4.2 - I think it would help to have a schematic picture of the moulin and the subglacial channel showing some of the various variables. It is slightly frustrating - but I can see that it may be unavoidable - to have the moulin shape model coupled so tightly to a subglacial channel model; ideally you'd like to be able to model the moulin separately. In this case, it seems that the subglacial channel is assumed to run from the bottom of the moulin to the ice-sheet margin, along which length the channel cross-section would presumably vary in reality, but I think that you assume a single value of  $S$  (the value at the bottom of the moulin?) is sufficient to describe how the flow evolves? This seems a reasonable simplification here, but I think could be explained a bit better, and as I say, a diagram might help. The 'b' in the hydraulic gradient on line 339 seems to disappear when this term is inserted in (28). The diagram might also help to explain  $Q_{base}$ ,  $Q_{in}$

and  $Q_{out}$ . The use of  $Q_{base}$  seems fine to me, as for most moulins there will likely be water arriving at the bottom of the moulin from upstream as well as via the moulin.

Section 3 - The results section focuses a lot on parameter sensitivity, and it is great that this has been explored so thoroughly, but I found this hard to follow without it having first been outlined some of the general behaviour of the model. In particular, I think it would be helpful to see some sort of figure showing the periodic states to which the moulin apparently evolves. Just the fact that the modelled moulin approaches an 'equilibrium' does not seem an obvious result, and I think that equilibrium could be described a bit more fully. Presumably it involves the water level moving up and down on a diurnal timescale, and the moulin opening and closing? It would also be useful to know how this depends on the moulin input  $Q_{in}$  (for me that would seem more of interest than dependence on drag parameters etc, which we don't know very well). It seems quite surprising to me that if such an equilibrium is really reached, it depends on the initial moulin radius. Also, has the moulin model been run over the course of multiple years (with melt season and a winter), and how does it behave? This has implications for what an appropriate 'initial' moulin radius is, presumably. I think it would be helpful to have some general discussion along these lines, and figure(s) (perhaps like figure 6 or 8) that show the general behaviour of the model, before going into detail about how certain outputs depend on the parameters, since it would help give those more context.

Figure 6 - see my earlier comments about comparing elastic and viscous deformation. I just don't understand what is actually plotted in panel f and g. Could you express whatever quantity is being plotted in terms of variables in the equations? Similarly for figure 9, and the associated discussion in section 4.5

More minor comments

L82 - why does taking  $k = 1$  approximate likely channelized pathways? The usual thinking is that channels would tend to \*lower\* the water pressure and would therefore be associated with a lower value of  $k$ , if anything.

Figure 1 is very nice. It might be noted that the elastic deformation here is quite different from all the other ones, in that the others are all \*rates\* - they accumulate every timestep to give continued deformation - whereas the elastic one is just static.

L185 - the small component of melting due to temperature differences between the water and ice seems to be ignored in the model, since it is later assumed that the water is at the melting temperature ?

L255 - you seem to use both hydraulic diameter  $D_h$  and hydraulic radius  $R_h$  and it would keep the notation simpler to just work with one or the other.

L262 - it sounds like in the end you take  $f_R$  to be fixed (and vary it's value) so I wasn't sure what the point of introducing (16) was.

In (18) presumably  $S$  is really  $A_m$ , the moulin cross-sectional area?

In (19) is  $dh_L/dz$  the same as  $dh_L/dL$  in (15), and is there significance in the change from lower case to upper case subscripts?

In (23), time appears to be in hours, not days.

In (24),  $h$  is the same as  $h_w$  ?

Figure 7 - should there be a purple line in panel (d)?

L662 - I wasn't able to see this statement about the fixed moulin frequently overtopping the moulin in Fig 11a. How does the figure show this?