Comment on tc-2021-192
Anonymous Referee #2

Referee comment on "Three-Dimensional Stefan Equation for Thermokarst Lake and Talik Geometry Characterization" by Noriaki Ohara et al., The Cryosphere Discuss., https://doi.org/10.5194/tc-2021-192-RC2, 2021

The discussion paper analyzes geometry of taliks forming below permafrost thaw lakes, compares the resulting model to field data from one site, and provides a more qualitative discussion on permafrost thaw lake dynamics and influence of those dynamics on talik growth. The work provides insights into environmental controls on thaw lake taliks and implicitly on their evolution in a warming Arctic. The subject is of significant interest to the readers of TC and the approach used is relatively novel and appropriate. Overall, the discussion paper provides some interesting insights and is a welcome contribution. I do think the manuscript could be improved by paying some more attention to the presentation of the model in Section 2 and to implications of the work.

Specific comments.

- The title of the discussion paper focuses on the analysis of Section 2, but doesn’t provide an adequate indication of Section 4, which provides a much richer picture of thaw lake and thaw lake talik dynamics. This paper might be more impactful if the authors can find a title that better reflects the breadth of the analyses.
- Description around the derivation in Section 2 is often unclear and in some places imprecise and will make it difficult to follow for some. For example, it’s clear in the scalar equation 1 that q_f is the energy available to thaw permafrost per unit time and area. Generalizing to the vector equation 5, it’s a little easier to understand q_f as the velocity of the moving phase boundary scaled by the volumetric latent heat of fusion for water ice to make it an equivalent heat flux, as in equation 2. Referring to q_f as the ‘fusion heat vector’ is a bit obscure. This might be easier to follow and would avoid that jargon by doing the analyses in the velocity v instead (eg. By dividing both sides of eq 5 by \phi \rho L see Eq 2). An alternative approach would be to clearly describe the physical interpretation of the vector q_f around Eq. 5, give q_f a better name and stick to that name in the rest of the manuscript.
- It would be helpful to summarize assumptions behind Eq 1 when Eq 1 is introduced. This is addressed somewhat in Section 4, but it would be helpful have that stated more explicitly.
- The reader needs to know why the functional F is introduced this way in Equation 15 (i.e. you want to minimize the boundary area for a specified thaw volume, the symbol
\(\lambda\) is a Lagrange multiplier, etc.) Also in line 197, it would be clearer to say “for a specified talik volume” instead “for the total talik expansion”. Similarly, the sentence starting on Line 194 could be clarified.

- This paper contains several insights that could inform representations of thaw lake dynamics in Earth System Models. If possible, it would be useful if the authors could comment on that.