

The Cryosphere Discuss., referee comment RC1
<https://doi.org/10.5194/tc-2021-192-RC1>, 2021
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Comment on tc-2021-192

Anonymous Referee #1

Referee comment on "A new Stefan equation to characterize the evolution of thermokarst lake and talik geometry" by Noriaki Ohara et al., The Cryosphere Discuss., <https://doi.org/10.5194/tc-2021-192-RC1>, 2021

This paper presents a neat mathematical approach to determining the shape and thaw rates of thermokarst lakes, with a comparison to field data. Overall a contribution I would be excited to see in the literature when some fairly minor issues are addressed and clarified.

General Comments as requested by TC:

- Does the paper address relevant scientific questions within the scope of TC?
- Paper discusses the mathematical representation of talik development around thermokarst lakes with predictive ability to represent thaw rate and talik shape. This is indeed a gap, and interesting to address.
- Does the paper present novel concepts, ideas, tools, or data?
 - Approach is relatively novel, a mathematical model used to represent this process is a good idea.
- Are substantial conclusions reached?
 - In my view, though the model is nifty and mathematically neat (except some small areas where further details/assumptions should be stated to ensure applicability), there

is essentially no field data that adequately corroborates the result. The lake used to 'validate' the model fits reasonably well along one depth profile, but the entire bathymetry for the lake is not represented, and it is stated that this lake may actually be composed of two different thaw features. It would be of great added value to compare the model to multiple lakes in an area where wind and uneven segregated ground ice distribution are not a large factor to see whether the lakes formed do indeed follow the model. Also to predict time series thaw in lakes as compared to model predictions. As it is, the comparison to data concerns me as it looks like validation, but the model would be more 'trustworthy' if stated in theoretical terms as opposed to in direct application.

- Are the scientific methods and assumptions valid and clearly outlined?
 - For the most part, though some of the derivation needs a bit of clarification; likely the results hold, but as a reader I struggled to follow some of the steps
 - Sometimes the assumptions previously stated should be reiterated (especially in the discussion) to ensure the readers understand the limitations of this model.
- Are the results sufficient to support the interpretations and conclusions?
 - Not really... as noted above, the model derivation is nice, but comparing the model results to one measured lake is a little worrying.
- Is the description of experiments and calculations sufficiently complete and precise to allow their reproduction by fellow scientists (traceability of results)?
 - As noted above, some detail in derivations is missing. The manuscript is also quite long, and so I was unable to review the supporting information as I would like and cannot comment on the quality of the derivations therein.
- Do the authors give proper credit to related work and clearly indicate their own new/original contribution?
 - Yes, though the introduction may need a few additional references for some 'obvious' concepts which are clearly not the author's own ideas, but generally accepted in the field.
- Does the title clearly reflect the contents of the paper?
 - Yes
- Does the abstract provide a concise and complete summary?
 - Yes

- Is the overall presentation well structured and clear?
 - Yes
- Is the language fluent and precise?
 - Some minor issues noted below, and some jargon and unnecessarily complex language used to describe especially mathematical derivations
- Are mathematical formulae, symbols, abbreviations, and units correctly defined and used?
 - For the most part, some missing units and inconsistent use of symbols detailed below
- Should any parts of the paper (text, formulae, figures, tables) be clarified, reduced, combined, or eliminated?
 - I am unsure of the comparison with field data - I think either this section should be expanded to include more sites, removed (which I am sure the authors agree would detract from the merit of this contribution), or perhaps re-phrased as an example application of this new method and not a test of the method proving its efficacy.
- Are the number and quality of references appropriate?
 - I believe so, apart from the needed citations in the introduction
- Is the amount and quality of supplementary material appropriate?
 - As noted above, due to the length and number of derivations in the manuscript proper I was unable to review the SI

Specific Comments

l 23 ... the Euler equation **and** the calculus of variations

l 24 an extremum of the functional -> a minimum of the energy associated with the functional description of the phase boundary (for clarity)

l 32 stabilizes thermokarst lakes -> stabilizes the size? shape? growth rate? be specific

l 45 ... above an unfrozen water body (CITATION NEEDED)

l 50 ... bed consequently subsides (CITATION NEEDED)

l 84 advanced the technique by including (parameter of) <-remove advective heat transport

l 141 k_L needs units!

l 151 should read: the letters in bold denote vectors.

l 152 remove 'also'

l 181 in the horizontal direction -> the horizontal gradient

l 182 this is not easily interpreted, especially for the general audience of this journal. Are you implying Stoke's theorem? I think it would strengthen the derivation to begin from a more 'certain' or understandable place than eq 183

eq 13 I think it may make more logical sense to present this in the opposite direction - the integral along the phase boundary (line) is not something that I can interpret easily or can be visualized, whereas something more like the flux across the phase change surface, or the volume integral of the total energy in the lake is more easily interpreted. I would start with the resulting equation and state which theorem (Stoke's?) is used to get the first equation. Importantly providing a physical interpretation (in more simple terms) of what each expression (start and derived result) means and how it is useful and what it tells us about the system. This would greatly increase the utility of the work for those who are

less interested in the mathematics and more interested in their application.

l 187 section heading is redundant: optimum phase boundary shape - is sufficient

l 190 "here, we present the ..." - present is not a good word choice

l 193 "weighted phase boundary area" this is not easily visualized/interpreted. Can you describe more concretely what this key phase means and what quantity is weighted along the phase boundary before using it (it occurs several times throughout)

l 195 it does not seem logically evident that the shape of a talik would preferentially minimize the total permafrost thaw given an amount of incoming energy. Please either provide a reference, explain the logic, or reconsider. The energy would simply flow according to the thermal gradients, and more energy would be used where gradients are strongest, no? If there is data for the temperature gradients here or in other lakes that would be helpful.

eq 14 the volume is a double integral and not a triple integral - why?

l 224 (and future occurrences) "this 3D Stefan equation" should it not be the **solution** to the 3D Stefan equation? Where do you introduce the Stefan equation? eq 4? State in the text when this occurs otherwise this conclusion to the methodology here doesn't really make sense

eq 24 - define units for new parameters, and define r_{deg} in text. Where is this equation from? Appendix? if so please say where to find derivations, if not consider including this.

l 245 unlikely -> not a

l 239-248 note if there was evidence of segregated ground ice in these sites

eq 25 r_{sub} what is this?? Is it the same as r_{deg} ?? rename or define

I 269 missing work : ... not eh western shore **determined** form....

ISSS 3.1 describe thaw rates a bit more (especially in reference to figure) the range is wide, I assume the high thaw rates are observed in a similar location... also how complex is the shoreline shape? Does it vary around the lake?

I 283 solution to ... also remove 'the' based on 27 talk thickness ..."

I 301 model geometry not geometry model

I 304 lake expansion is most rapid

I 324 "wind erosion effect" not mentioned until now, please elaborate earlier or save this for the section reporting it exclusively

I 341 is the assumption that the radial thermal gradient is zero accurate? Other publications report much more rapid than vertical thaw (though my focus is discontinuous PF) see McClymont et al. Devoie et al. work at Scotty Creek. Please cite something or report thermal gradients to support this

I 371 the preceding discussion all hinges on the zero lateral gradient assumption - please highlight this otherwise it seems unlikely

I 395-396 this is the first mention of anthropogenic processes, and none of the bullet points are direct human activities. Suggest to remove this idea unless there is an additional section on anthropogenic change

I 408 what are horizontally oriented lakes? Please describe as this term is not clear

I 443 easier to understand would be: Wind-driven wave action make the water bodies round.... (remove asymptotically - this does not belong)

I 493 why is it more rapid?

I 510 for the horizontal stage (COMMA), A in figure 6,

I 527 solution to 3D Stefan equation is limited

I 535 what about anisotropic thermal properties? Maybe also discuss these as well?

I 541 state the thermal effect on thermokarst morphology, I am not sure what this refers to in the manuscript as is written now, so it is either unclear or unsupported

I 557 weighted phase boundary (again weighted according to what?)

Suggestion: due to the lack of evidence supporting the conclusions on shape, it seems that the argument is stronger for the phases of formation and evolution of thermokarst lakes, so it may be more relevant to report more strongly on this aspect in the abstract and conclusion? As an alternative/addition to my previous comments on data comparison.