Reply on RC2

Duan Li et al.

Author comment on "Interpolation of magnetic anomalies over an oceanic ridge region using an equivalent source technique and crust age model constraint" by Duan Li et al., Solid Earth Discuss., https://doi.org/10.5194/se-2021-117-AC1, 2022

We are also grateful to the reviewers for the assessments, comments, suggestion and recommendations. All of them are carefully considered while revising the manuscript. Below we provide a point-by-point response to all pieces of suggestion and comments.

Reviewer #1:

The manuscript proposes a space-domain method that uses a pair of equivalent layers for interpolating sparse total-field anomaly data on the oceanic crust by using an age model as a constraint. Although not clearly specified in the manuscript, the method is developed in a topocentric Cartesian Coordinate system with x, y, and z axes pointing, respectively, to North, East, and down. The method consists in solving a constrained-linear inverse problem for simultaneously estimating the physical property distribution on the two layers that yields an acceptable total-field anomaly data fit. The method imposes smoothness along isochrons of oceanic crust only on the physical property distribution of the shallow layer with the purpose of filling the gaps of total-field anomaly data. For me, the manuscript needs to be significantly improved before being considered for publication. The main problems are listed below:

(1) The equivalent-layer technique is offered as a better alternative to kriging, minimum curvature, cubic spline interpolation, and inverse distance weighting methods for interpolating sparse total-field anomaly data on oceanic crust because "these methods might not be optimal for the data prediction in areas with insufficient data" (page 4). The problem here is that the equivalent-layer technique is also negatively affected by insufficient data.

Reply: We agree with this comment. Insufficient data is a limitation for any method, and our work is to compare the accuracy of each method for data prediction in the same situation. The equivalent source (ES) method is to transform observed data into source, and then make data prediction through the source. Thus, the ES method is better in physical principle than the method based on morphological characteristics of data, and the calculation results also support the conclusion.

(2) At the end of page 4, beginning of page 5, it is written that the equivalent-layer technique may provide a more accurate magnetic field because it is possible to improve its structure and distribution. In my opinion, this justification should be considerably improved. It is not clear how the structure and distribution of the equivalent layer can be

modified to produce a more accurate field at the interpolating points. I understand that, by increasing the number of sources composing the equivalent layer, it is possible to obtain an exact data fit at the observation points because the inverse problem becomes underdetermined.

Reply: It is an expression of the research result of Li et al. (2020, GRL), indicating that the calculation accuracy can be effectively improved by improving the structure and distribution of the ES. Therefore, in our work, a similar technique is expected to achieve better interpolation result, which has also been proved in the synthetic model test.

(3) The proposed method uses the crustal age model of Müller et al. (2008) as a priori information for constraining the linear inversion of total-field anomaly data on oceanic crust. This model, in turn, was obtained on the basis of marine magnetic anomaly identifications. It seems that there is a circular reasoning problem here. Because the age model depends on the magnetic data, it does not necessarily introduce new information into the inverse problem and apparently cannot be used as a constraint.

Reply: The crustal age model is only used to provide constraints on the direction trend, so that the equivalent source could extends in a specific direction, which does not affect the data fitting. What we provided in this work is a constraint method or idea. In addition to crustal age, other directional constraint information can also be converted into the weighting factors to participate in the inversion. Moreover, we want to recover the magnetic anomaly field which is helpful to construct the global lithospheric magnetic field, such as the EMAG2v3 (Dyment et al., EPSL, 2015; Lesur et al., EPS, 2016) and WDMAMv2 (Meyer et al., G3, 2017). These models also used the crustal age model of Müller et al. (2008).

(4) Matrices **Wx** and **Wy** (eq. 3) impose smoothness along x and y directions. However, the isochrons are not necessarily aligned with x or y directions. So, it is important to clearly explain how the proposed method deals with isochrons that are not aligned with the x or y directions.

Reply: Since the isochron or boundary of lineation is discretized and corresponds to the equivalent-source cell one by one. Whether the lineation is aligned with the x or y direction, large values of w_x or w_y are taken for cell inside the lineation, small values of w_x or w_y are taken for cell at the boundary of lineation, or small values are taken for both w_x or w_y.

(5) The simulated crust (Figure 1) has isochrons that are perfectly aligned with the North-South direction (x-axis). In this case, matrix **Wx** (eq. 3) can be used to impose a strong smoothness along the x-direction. However, this model represents a very ideal situation. The simulated survey lines (Figure 2) are perfectly orthogonal to the simulated isochrons. This is also a very ideal situation. For me, the test with synthetic data presented in Section 3 should be used as an initial validation test. The conclusions obtained from this test cannot fully support the interpretation of real data. In my opinion, additional tests with synthetic data produced by models reproducing or at least approximating the complexity of a real magnetic survey on oceanic crust should be included in the manuscript.

Reply: We agree with this comment. As we answered in the last question, the constraint principle is the same regardless of whether the lineation changes are complex or not.

I have also some specific comments/recommendations:

(6) It seems that the method uses a topocentric Cartesian system with x pointing to North, y to East, and z pointing down, but I could not find this information in the

manuscript.

Reply: Thank you for the suggestion. We have defined the coordinate system in the manuscript.

(7) On page 6 is written that "Regularization and precondition techniques were utilized to stabilize the inversion process and balance the decay of the potential field". I understand that a preconditioning technique, in this case, does not introduce a priori information about the parameter vector **m** (eq. 1), but only controls the convergence. So, could you please explain what is the a priori information introduced by matrix **P** (eq. 1) and how it contributes to stabilizing the inversion?

Reply: The introduction of **P** is in the supplementary materials for the manuscript, which was uploaded simultaneously. The diagonal element of **P** is z^{β}, where z is the central depth of the ES cell and β is the weighting index, which can be determined based on the attenuation characteristics of the potential field generated by the corresponding ES cell (e.g., Liu et al., 2015; Li et al., 2020).

(8) I think that the elements forming matrices **G** and **P** (eq. 1) must be clearly defined in the manuscript. Note that, without specifying the elements of matrix **G**, the reader cannot know what type of equivalent sources (prisms, dipoles, etc) form the equivalent layer.

Reply: Thank you for your suggestion. The element forming matrices **G** and **P**, and the type of equivalent sources are introduced in the supplementary materials for the manuscript, which was uploaded simultaneously.

(9) I recommend using a tool model to illustrate how matrices **Wx** and **Wy** (eq. 3) are defined.

Reply: Thank you for the suggestion. The definition of $\mathbf{W_x}$ and $\mathbf{W_y}$ are illustrated in the supplementary materials for the revised manuscript.

(10) According to page 7, matrices **Wx** and **Wy** (eq. 3) impose smoothness only on the physical property distribution of the shallow layer. Why they are not used to also impose smoothness on the deep layer?

Reply: The deep layer of equivalent source is utilized to simulate the background magnetic field, which is unpredictable in practice. Therefore, the deep layer is not constrained. We have added relevant explain in the revised manuscript.

(11) What are the criteria to define the depth/geometry of shallow and deep layers?

Reply: The shallow layer is placed in the depth range of 1~6 times the observed data spacing, which is the previous research experience. The depth of deep layer is determined by the logarithmic power spectrum of observed data. We have supplemented to the paragraph in the revised manuscript.

(12) On page 7, it is written that "A layer with larger ES cell sizes at larger depth was utilized to simulate the background magnetic field.". I understand that "changing cell sizes" is possible only if the layer is formed by 3D sources. How to change the cell sizes of a layer formed, for example, by dipoles?

Reply: In this work, we adopted the prism as the cell to construct the dual-layer equivalent source, which is illustrated in the supplementary materials for the revised manuscript. Therefore, the geometric size of the cell can be changed.

(13) Apparently, the weights w_x and w_y (elements of matrices **Wx** and **Wy**, eq. 3) do not have any normalization. In this case, it is expected that their numerical values depend on the particular characteristics of the study area and the interpretation model. As a consequence, it is not possible to use a fixed 10^-4 in all situations. I recommend including some discussion about this.

Reply: The values of w_x and w_y are still based on experience at present, and we have not been able to work out a quantitative setting standard. In fact, these values only serve to increase the difference of magnitude between weighted and un-weighted cells. In our experience, 10^4 or 10^{-4} can generally work. If the effect of this setting is not obvious in the real data application, the values of w_x and w_y can be increased according to the change of calculation results.

(14) What is the "geophysical meaning" of the synthetic magnetic interface presented in Section 3? Could it be related to the Curie isotherm? In this case, I think it should be smooth. It seems that this simulated magnetic interface is a purely mathematical way of generating long-wavelength data.

Reply: The background field is simulated by magnetic interface with random fluctuation, in order to simulate the unpredictable long-wavelength information in practical. In some cases, long-wavelength information is more than just Curie fluctuations. If the calculation accuracy of method can be guaranteed in this case, the processing effect may be better if long-wavelength is simpler.

(15) The simulated main geomagnetic field presented in Section 3 is constant, with intensity, inclination, and declination of 35000 nT, 40°, and 3°, respectively. The crust model, however, covers an area of approximately 5° × 5°. Is it reasonable to consider that the main field is constant throughout this area?

Reply: In our work, the IGRF model was used to obtain the intensity at each data point, and the magnetic anomaly was obtained by subtracting the intensity. The magnetization of equivalent source is obtained by the inversion of magnetic anomaly. The average values of inclination and declination of the study area were used to the calculation. According to our previous works, using constant inclination and declination has a great influence on reducing to the pole, but the influence on interpolation and continuation can be ignored. In further work, we will use the variation inclination and declination in the calculation to improve the calculation process.

(16) In my opinion, a detailed description of the parameters used to generate the results shown in Figure 2 with all methods must be included in the manuscript. Otherwise, it is not possible to obtain a proper comparison.

Reply: When using other methods, the parameters selected are common and default. In addition, the directional constraint is also considered in the calculation of these methods, so we think it is comparable.

(17) The study area in Section 4 covers an approximately 5° x 5° area. Is it reasonable to consider that the main geomagnetic field is constant throughout this area? How the variability of the main field affects the results?

Reply: Thank you for the question. As the answered in question (15), the IGRF model was used to obtain the intensity at each data point, and the magnetic anomaly was obtained by subtracting the intensity. The magnetization of equivalent source is obtained by the inversion of magnetic anomaly.

(18) I think that Figure 3 should be improved. I could not understand the relationship

between the axes "Northing" and "Distance" in panel (b). Apparently, panel (e) shows the two layers, their equivalent sources, and the weights w_x and w_y (elements of matrices **Wx** and **Wy** in eq. 3) associated with them, but it is not clear for me.

Reply: The main purpose of using "distance" on the right side of the figure is to draw each survey line separately. The "distance" is the distance between other measuring points and the southern vertex of each survey line. Panel (e) shows the value and distribution of the w_x and w_y of each equivalent source cell. We also tried other forms of expression, but we think the proposed design is easier to convey more information by compared these forms.