Reply on RC1
Vladimir Gnevyshev and Tatyana Belonenko

Author comment on "Analytical solution of the ray equations of Hamilton for Rossby waves on stationary shear flows" by Vladimir Gnevyshev and Tatyana Belonenko, Ocean Sci. Discuss., https://doi.org/10.5194/os-2022-5-AC1, 2022

Dear Reviewer,

We thank the anonymous reviewers for carefully reading the manuscript. Please find below a detailed point-by-point response to all comments.

- The main issue I have with this work is that it teaches us nothing new about "Ocean Science". The authors make no attempt to relate the solutions they compute to an oceanographic phenomenon. I'm not sure whether this work warrants publication as a contribution in applied mathematics or fluid mechanics but it is definitely not a contribution in oceanography. The authors do not attempt to contribute anything to our understanding of phenomena observed in physical oceanography.

The next step in the study is to find a specific application to ocean processes using satellite, ground and model data. At this stage, we show that there is a qualitative difference between the problems for zonal and nonzonal background flow and confirm this with examples.

- From a mathematical standpoint, it is rare for nonlinear, 4th order, systems to poses analytic solutions. However, for the most part, numerical solutions are both easy to compute and very accurate (unless there is a singularity in the problem which is not the case here). The ray equation is no more than the calculation of trajectories in space when the (group) velocity is spatially variable (including the spatial variability of the wavenumber). It is unclear what has been gained from the few trajectories calculated in this manuscript. The main message of this paper is too thin mathematically. As highlighted by the authors, the attraction of all trajectories to the critical point (e.g. latitude) is well known and was highlighted in previous papers by the same authors (Gnevyshev et al., 2020a; 2020b). Solutions of such basic mathematical systems are publishable only when they constitute significant contributions to our understanding of oceanographic processes.

You caught the main physical idea - this is the extreme sensitivity of Rossby wave tracks. But we wanted to say that such sensitivity exists only for non-zonal currents. Tracks on the zonal flow have a continuous dependence on the initial conditions and in this sense, the zonal task is extremely predictable.
When you discuss about “...unless there is a singularity in the problem which is not the case here etc.” you’re mistaken. The fact that the Rossby baroclinic waves have an extremely unpleasant singularity on the hyperbolic Booth lemniscate is obtained in the works of Longuet-Higgins. The intersection point of the lemniscate with the abscissa axis was also highlighted by Pedlosky [13]. Apparently, we need to write a separate paper on this topic to explain the specifics of Rossby waves as a system consisting of two types of waves: waves with positive and waves with negative dispersion. The hyperbolic lemniscate is the boundary of these types of waves. On the lemniscate there is a singularity of Rossby waves, as at the boundary of the change of the sign of the dispersion of Rossby waves.

- The description of the underlying assumptions and the methodology employed is very poor. The extremely weak pedagogical presentation typifies both the English style and the mathematical analysis. If the authors wish to publish this work in a different journal they should tend much more seriously to both of these aspects of communication with their readers. A few examples are listed below as an aid to the authors if they intend to submit the manuscript to another journal.

We took into account all your comments and improved the text. Once again, we want to thank you for your help.