Reply on RC3
Peter Chu

Thank you very much for reviewing my manuscript.

Response to the Geopotential Surface and Vertical Direction

“I have read the manuscript by Peter Chu, and while I found it quite thought-provoking, I am forced to conclude that it is actually quite misleading and do not recommend publication in its present form. It seems to me that the mistake the author is making is to formulate the equations of motion in spherical coordinates from the beginning. This is not my understanding of how the equations of motion used by models of the atmosphere and ocean are formulated. Rather, these models use a coordinate system in which the vertical direction is defined as being perpendicular to geopotential surfaces so that gravity always points along the vertical direction with no horizontal component.”

I disagree.

Oceanographers have used the geoid for several decades, but almost no one recognizes that the geoid surface represents the true horizontal. Perpendicular to the geoid surface is the true vertical. The geoid surface is a TRUE GEOPOTENTIAL SURFACE at the top ocean. The global geoid data are publicly available from gravity models of geodetic community such as the EIGEN-6C4, which shows that the geoid surface varies from -106.2 m to 85.83 m.

However, the geopotential and geopotential surface used in oceanography and meteorology are the normal geopotential and normal geopotential surface, but not the TRUE GEOPOTENTIAL and TRUE GEOPOTENTIAL SURFACE.

The two attached figures illustrate the difference between the normal gravity which is called the effective gravity and used in oceanography and meteorology, and the true gravity which is the most important variable in geodesy.

Figure A shows the main features of the effective gravity [-g(φ)K]: (1) it is determined from the solid Earth with rotation and uniform mass density; (2) the unit vector K is perpendicular to the z surface (z = constant) and points the normal vertical; (3) the z surface is the normal horizontal and coincides with the normal geopotential surface; (4) any movement on the z surface (i.e., normal geopotential surface) is not against the
normal gravity.

Figure B shows the main features of the true gravity \( g(\lambda, \phi, z) = -g(\phi)K + \delta g \): (1) it is determined from the solid Earth with rotation and non-uniform mass density; (2) the true gravity has never been used in oceanography and meteorology; (3) the true gravity vector \( g(\lambda, \phi, z) \) is perpendicular to the true geopotential surface such as the geoid surface, which represents the true horizontal; (4) any movement on the true geopotential surface is not against the true gravity; (5) any movement on the z-surface is against the true gravity. An additional force, the gravity disturbance, shows up in the z-surface momentum equations.

**Response to the Coordinate System**

"The resulting coordinate system is orthogonal and curvilinear with the horizontal surfaces varying in distance from the centre of earth in response to variations in the geopotential. The usual practice in the modelling community is to approximate this curvilinear coordinate system by spherical coordinates. It seems to me that the onus is on the author to show that the terms that are neglected when this approximation is made are important and should not be neglected. It should be noted that starting from an orthogonal, curvilinear coordinate system in which the horizontal surfaces correspond to geopotential surfaces, and then approximating the resulting system of equations using spherical coordinates, is not the same as formulating the governing equations in spherical coordinates from the beginning, as the author insists on doing."

I disagree.

The ocean dynamics to include the effect of the gravity disturbance should be coordinate independent. I use the vector form to redrive the combined Sverdrup-Stommel-Munk equation (see Supplement).

**Response to the Hydrostatic Balance**

"It is worth noting that I have no argument with equation (12) in the manuscript which is written in vector form. One consequence of this equation is that the equilibrium state is the one in which isopycnal surfaces coincide with geopotential surfaces, as implied by equation (12) when the pressure gradient term is balanced by the term involving gravity, corresponding to hydrostatic balance. In the coordinate system used by ocean modellers, the equilibrium state corresponds to horizontally uniform stratification."

"I also feel that the author is being too relaxed in his treatment of the hydrostatic approximation since this strictly applies only in the coordinate system in which gravity acts in the vertical direction. However, these are minor points compared to what I have written in the first paragraph of my review."

The vertical direction depends on the gravity. The vertical is in the z-direction for the normal (effective) gravity (shown in Fig. A of the Supplement), and in the direction normal to the true geopotential surface such as the geoid for the true gravity (shown in Fig. B of the Supplement). Since normal gravity is just an approximation of the true gravity, replacement of the normal gravity by the true gravity in ocean dynamics becomes necessary.

Equation (15) is the hydrostatic equilibrium with the true gravity. I explain it clearer in the Supplement.

**Response to the Data Format**
"Another issue I have with the manuscript is the way in which the author evaluates the Jacobian term in his equation (19) using data directly from the World Ocean Atlas without comment. At the very least, one needs to ask what coordinate system is being used by the World Ocean Atlas and whether this is the same as the coordinate system being used in equation (19). Indeed, is it appropriate to simply insert data from the World Ocean Atlas directly into the Jacobian operator?"

I agree. Both geoid ($N$) (from the EIGEN-6C4) and in-situ density data ($\rho$) (from WOA18) are represented in spherical coronates. The spherical coronates are only used to estimate the gravity disturbance forcing (GDF).

**Response to Your Recommendation**

"In summary, I cannot recommend publication of this manuscript in its present form and I believe the argument being put forward by the author is flawed. At the very least, the author needs to formulate the governing equations in the orthogonal, curvilinear coordinate system in which gravity always acts in the vertical ($z$) direction. He then needs to consider the terms that are neglected when this coordinate system is approximated by spherical coronates. Since these terms will, at best, involve accelerations terms arising from the curved surfaces that correspond to geopotential surfaces, I cannot see that these terms can be important or that models, as currently formulated, are fundamentally wrong. If the author believes otherwise, the onus is on him to show readers what these terms are, and why they are important."

The ocean dynamics to include the effect of the gravity disturbance should be coordinate independent (see Supplement).

The spherical coordinate system is used because the geoid and density data are represented in the spherical coronates.

After reading my responses, you may change your recommendation.

Please also note the supplement to this comment: