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## **Comment on npg-2022-3**

Anonymous Referee #1

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Referee comment on "Effects of rotation and topography on internal solitary waves governed by the rotating Gardner equation" by Karl R. Helfrich and Lev Ostrovsky, Nonlin. Processes Geophys. Discuss., <https://doi.org/10.5194/npg-2022-3-RC1>, 2022

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The review of the manuscript

"Effects of Rotation and Topography on Internal Solitary Waves Governed by the Rotating-Gardner Equation"

by Karl R. Helfrich and Lev Ostrovsky

This referee was asked to join the discussion at the late stage, when the previous comments were no longer visible on the system. Hence, my comments are completely independent of the previous discussion which I have not seen.

The paper continues the line of research on internal waves within the scope of the KdV-type models. This study is based on the use of the variable-depth, rotating Gardner model, i.e. it aims to highlight the effects due to the common action of nonlinearity (including cubic nonlinearity), dispersion, rotation and depth inhomogeneity. Analysis is based on the previously developed adiabatic theory (with some corrections), and is compared to the results of direct numerical simulations of this model equation using a pseudospectral method. The results presented in a number of high-quality Figures look instructive and convincing, showing good agreement for the case of weak nonlinearity, and clarifying the limitations of the adiabatic theory when the amplitude parameter is increased.

In the view of this referee, the following points need to be clarified in order to put the research in the context of other studies:

1. Could you, please, provide a reference to the derivation of this model equation from the primitive equations. Can it be derived in a systematic way? Does this model include all terms appearing at the relevant order of the amplitude parameter?

If not, please, explain why and when we can disregard some terms appearing in the systematic asymptotic derivation (e.g., the 5th derivative and nonlinear-dispersive terms in the extended KdV equation). I suspect that there could be some additional terms in the case of variable topography and rotation.

Also, please, compare to the results in this paper and discuss in that context:

A. Karczewska, P. Rozmej, Can simple KdV-type equations be derived for shallow water problem with bottom bathymetry? *Comm. Nonlin. Sci. & Numer. Sim.* 82 (2020) 105073.

2. The results of the study need to be compared and contrasted to the two recent papers by Yury Stepanyants:

- Yu. A. Stepanyants, The effects of interplay between the rotation and shoaling for a solitary wave on variable topography, *Stud. Appl. Math.* 142 (2019) 465-486.

- Y.A. Stepanyants, Nonlinear waves in a rotating ocean (the Ostrovsky equation and its generalizations and applications), *Izvestiya, Atmospheric and Oceanic Physics*, 56 (2020) 20-42.

In particular, in these studies it was shown that for solitary waves moving towards shallower waters, the terminal decay caused by rotation can be suppressed by the shoaling effect. Is this result confirmed in your studies?

3. This sentence in the Introduction is only partially correct:

"It is also known that rotation destroys internal solitons due to resonant radiation of inertia-gravity waves (terminal damping; see Grimshaw et al., 1998a)."

This is true in the absence of currents. However, the sign of the rotation coefficient in the Ostrovsky equation may be changed by the underlying shear flow, and then the equation supports solitons, see the first examples in

A. Alias, R.H.J. Grimshaw, and K.R. Khusnutdinova, Coupled Ostrovsky equations for internal waves in a shear flow, *Phys. Fluids* 26 (2014) 126603.

4. Lines 19-20, should it be 'specific case'?

To summarise, assuming that the model equation can be justified, the study makes perfect sense, and the results are convincing. The validity of the model equation is not entirely clear and needs to be clarified by putting the research in the context of some other studies in the field, as suggested above.