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Comment on npg-2021-9

Anonymous Referee #2

Referee comment on "Identification of linear response functions from arbitrary perturbation experiments in the presence of noise – Part 1: Method development and toy model demonstration" by Guilherme L. Torres Mendonça et al., Nonlin. Processes Geophys. Discuss., <https://doi.org/10.5194/npg-2021-9-RC2>, 2021

Report on "Identification of linear response functions from arbitrary perturbation experiments in the presence of noise: Part I. Method development and toy model demonstration" by Torres Mendonça et al

The authors suggest a numerical method that estimates the linear response using only a single trajectory of the perturbed system together with a time series of the unperturbed system. Abandoning ensemble averages the usual response formula acquires an additional noise term. Their method consists of several steps. Firstly they introduce standard Tikhonov regularization. The regularization parameter is chosen to match the (unknown) noise level. In a second step the noise level is determined from the control trajectory of the unperturbed system. The authors discuss several additional ways how to optimise the regularization parameter by carefully determining what they call high and low frequency contributions and using a possible monotonic behaviour of the response function.

The authors perform careful and detailed numerical analysis of a simple toy model. They compare their results with existing methods based on ensemble averages. I particularly liked that they carefully discussed and illustrated the range of validity of their method and showing under what conditions it can be broken, for example by studying the addition of nonlinear effects.

I believe that this is a valuable piece of research that will be beneficial for the community, and I recommend publication after some minor issues have been addressed. I list the following comments the authors may want to incorporate:

** in the introduction the authors state that typically methods estimating the response rely on some "prior information". They state that their method does not need prior information. That seems not quite true (they assume monotonicity, Eqn (8), Picard condition etc).

** following up on my previous point, their method requires a few assumptions on the underlying dynamical system under consideration such as (8) and what they call the Picard condition. It would be nice to have these assumptions listed somewhere.

** it is well known that one can formulate regularization such as ridge regression in a Bayesian framework where the regularization corresponds to a prior. Could the

authors comment on what the meaning of this prior is in their context?

** introducing the noise η in (5) can be justified by the central limit theorem, I assume, which could be mentioned.

** although I appreciated that the authors went through some trouble in explaining the details necessary to understand their approach, the manuscript could gain by being more succinct. For example, the sentence right after Eqn (24) just reiterates what has been described before.

** in the introduction the authors give a nice account of the use of linear response theory in the climate sciences. Their exposition, however, might give the false impression that linear response should be expected. Whereas it is now proven that systems driven by noise satisfy linear response theory (Hairer and Majda 2010), the situation for deterministic systems as initiated by Ruelle is far more complicated. Viviane Baladi and co-workers in fact showed that very simple dynamical systems such as the logistic map do not obey linear response. Moreover, examples of dynamical systems in the climate sciences are known that exhibit a rough parameter dependency (Chekroun et al 2014). The question of how to reconcile the fact that generic high-dimensional dynamical systems satisfy linear response theory even when their individual microscopic constituents do not, was addressed by Wormell and Gottwald (2018, 2019). The following references are relevant for this discussion:

Chekroun et al, PNAS 111 (2014), 1684-1690

Hairer and Majda, Nonlinearity 23 (2010), 909

Baladi and Smiana, Nonlinearity 21 (2008), 677-711

Baladi and Smiana, Ergodic Theory Dyn. Syst. 30 (2010), 1-20

Baladi, "Linear response, or else," in ICM Seoul 2014, Proceedings, Volume III (ICM, 2014), 525-545, (e-print arXiv:1408.2937)

Baladi, Benedick, and Schnellmann, Invent. Math. 201 (2015), 773-844

Wormell and Gottwald, J Stat Physics 172 (2018), 1479-1498

Wormell and Gottwald, Chaos 29 (2019), 113127

** there are other recent methods dealing with response theory from a numerical point of view, either detecting it or calculating the response, which the authors may want to include:

Gottwald, Wormell and Wouters, Physica D 331 (2016), 89-101

Ni and Wang, Journal of Computational Physics 347 (2017), 56-77

Ni, "Approximating linear response by non-intrusive shadowing algorithms" (2020), arXiv:2003.09801 [math.DS]

** page 15, after Eqn (31): "Since almost every linear system can be diagonalised, we assume" → "We assume .."

** page 17, l439, delete "extremely"