General Comments:

This manuscript presents a new multivariate extension of the standard Gaspari-Cohn localization function which is compared with 3 other multivariate functions, plus their univariate versions. These techniques are extremely relevant to problem of cross-domain localization in strongly coupled data assimilation and this work is an encouraging step towards developing appropriate methods for such systems. The localization techniques are illustrated using the bivariate Lorenz 96 system.

Whilst it would be nice to have an example illustrating how the new multivariate GC method translates to a more realistic system I appreciate that it is always important to test new ideas like this in a relatively simple system, where results can be more easily interpreted. I hope that the authors have the opportunity to extend this work to a more complex coupled model system in the future.

It is always good to see new work addressing issues related to the application of coupled DA. The article is timely, highly relevant and clearly written; it will make a nice contribution to the coupled DA literature. I suggest it is published after minor revisions.

Specific comments:

- It is a shame that results were only shown for case where the fast (Y) component is fully observed, and further that the performance of each method was only measured/reported in terms of the RMSE of the X (slow, unobserved) component. I would like to see some results from experiments where only the X (slow) component is observed, and also where both the X and Y components are observed, both fully and partially. I appreciate that this would potentially increase the number of figures/length of the manuscript, but it may not be necessary to explicitly show all the results. A brief discussion of the results in order to confirm that the general conclusions still hold under different observing scenarios would give the reader greater confidence in the performance of the new GC method.

- I am not entirely clear on how the univariate localization functions were implemented. Lines 181-182 state:
"We compare the four multivariate localization functions in Sect. 2 to a simple approach to localization in coupled DA, which is to use the same localization function for all model components. We call this approach univariate localization."

I think this means that each block of the localization matrix \( L \) uses the same localization function and radius for all blocks, rather than a different radius for the X and Y blocks and a different function and radius for the cross X,Y block, is this correct? I think what is confusing is that you are calling it univariate localization but you are actually localizing the cross XY blocks of the matrix \( B \). Perhaps this needs to be stated more explicitly somewhere. In systems with very different error correlation scales this type of univariate localization function could be not really be expected to perform well.

**Minor comments:**

- The references are a bit strange – there are multiple web links for a lot of the papers; the https://doi.org/xxx link will be sufficient in most cases.
- Further minor comments and technical corrections are marked in the attached pdf.

Please also note the supplement to this comment: https://npg.copernicus.org/preprints/npg-2021-8/npg-2021-8-RC3-supplement.pdf