Nonlinear Processes in Geophysics Discussions

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Comment on npg-2021-25

Anonymous Referee #2

Referee comment on "Inferring the instability of a dynamical system from the skill of data assimilation exercises" by Yumeng Chen et al., Nonlin. Processes Geophys. Discuss., https://doi.org/10.5194/npg-2021-25-RC2, 2021

Inferring the instability of a dynamical system from the skill of data assimilation exercises
By Chen et al.
In this article, the authors present a theoretical upper bound for the analysis error of a chaotic dynamical system under perfect and linear model conditions. This upper bound considers the properties of the observation network (Spatio-temporal distribution of the observations) as well as the dynamical properties of the system. Then, the authors evaluate the relation between analysis errors and different observing networks and dynamical properties using an extension of the Lorenz 96 system. The results are quite interesting and show the impact of observing different variable types and changes in the system's dynamics. The authors also show that the proposed upper bound for the analysis error holds for this system under weakly non-linear regimes and that the magnitude of the analysis error is linked to fundamental dynamical properties such as the leading Lyapunov exponent or the Kolmogorov-Sinai entropy.
The paper is well written. The discussion of the motivation, methodology, and results is clear. I have only some minor comments and questions for the authors.

It would be interesting to add a discussion about how these results could change in the presence of model error. At least some hypotheses (like the convergence of P_f to the unstable subspace) may not hold in this case. Some discussion is included about the parametric model error, but also structural model errors are important.

L473 In this paragraph an idea on how to use DA to estimate \lambda_1 or \sigma_{KS} is presented. Results suggest that this is possible but requires investigating the behavior of the system under different dynamics. Can we estimate \lambda_1 if we have only one DA system with a particular observing network (like in operational DA)? Can the relations obtained for this particular system be extended to other systems?

L474 It is stated that using the analysis error and n_0, an estimate of \lambda_1 can be obtained. This is unclear for me. The relation between \lambda_1 and the analysis error seems to be empirically obtained in this paper. Are the authors assuming that the analysis error is equal to its upper bound (which is theoretically linked with \lambda_1 and n_0)?

L26 Kolmogorov-Sinai entropy (or metric)

L26 and can be identified as?

L83 asymptotic unstable-neutral modes?

Equation 7, please check the correctness of this equation.

L227 Please revise the definition of the potential energy (the summation index and the definition that should include \theta)

L261 signifies

Figure 1 Does this figure corresponds to the fully observed case?

L267 The last 500 DA cycles or model time units?

L285 very low values?

L292 Please revise the sentence starting with "Although these processes ..."

Caption Figure 3: ... where all variables, or only X, or only \theta are observed? Also describing each color line in the caption would be better. Also for Figure 5.

L319 The description of Figure 4 is unclear. Variable can refer to a grid point or different variable types. Maybe better to say variable type instead of just variable.

Figure 4: Are the variables normalized before the respective CLV amplitude is computed? Results, in this case, are reasonable, but I wonder how this analysis can be extended to more complex systems with variables with different ranges of variability and possibly different units.

Figure 7: In the caption, it is not clear if \sigma_{KS} represents the stable configurations or the weakly unstable model configurations.

L388 errors normalized?