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Nonlinear Processes in Geophysics
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Interactive comment on "Application of Lévy Processes in Modelling (Geodetic) Time Series With Mixed Spectra" *by* Jean-Philippe Montillet et al.

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Dear Reviewer,

Thank you for your comments. We would like to give a few answers to some of your comments. The modifications that we intend to do directly to the manuscript are highlighted with (R).

Major comments

Âń Can you explain a bit more systematically how (and why) different types of non-stationarity are associated to the different components of your stochastic model? Âż

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The stochastic noise model is basically divided in 2 components white + coloured noise. The coloured noise results from various parameters during the processing of the GNSS observations such as the mismodelling of GNSS satellites orbits, Earth orientation parameters, large-scale atmospheric or hydrospheric effects . . . (see Williams 2003, Williams 2004, Klos et al., 2018). Several studies in the past 2 decades have advocated that the coloured noise component is best described as a power-law noise, and particularly as a Flicker noise (power-law exponent = 1). A full discussion can be found in Montillet and Bos, 2019 (chapter 2).

(R): To recall why the coloured noise is modelled in the stochastic noise model, we will add the sentence "The coloured noise results from various parameters during the processing of the GNSS observations such as the mismodelling of GNSS satellites orbits, Earth orientation parameters, large-scale atmospheric or hydrospheric effects . . . (see Williams 2003, Williams 2004, Klos et al., 2018)."

- About the length of the time series

As we mentioned in the manuscript, there are several stochastic noise models which have been used to model the properties of the power-law noise (and in particular the Flicker noise), it includes FOGM, GGM . . . see line 25-30. The coloured noise is slowly varying, therefore the length of the time series is an important parameter in order to model this type of noise.

(R): The text will be modified accordingly (see also the discussion in the minor comments).

- "I am wondering about non-stationarity beyond just the mean, i.e., possible time-dependence inthe variance (or even higher-order distributional characteristics), which I believe can-not be captured by the present model setup but would require either some multiplicative component (e.g., functional model times noise of a certain type) or a stochasticmodel component beyond ARMA/FARIMA that inherets the property of conditional heteroscedasticity "

Here, we are only interested in adding a third stochastic processes. Several studies (e.g. Langbein 2008, Davis et al., 2012, He et al. 2019) have used the addition of a random-walk component in their stochastic noise model (i.e. White noise + Flicker noise + random-walk). as recalled in line 134-135. The aim is to model small transient signals (e.g. short post-seismic relaxation), residual signals (e.g., due to the non-deterministic nature of the seasonal signal), small offsets (buried in the noise floor) . . . . The sum of all these small amplitude transient signals is included in the definition of our residual time series (line 400-403). The validity of using the random-walk is generally justified in tectonic active areas (Langbein and Svarc, 2019; He et al., 2019). However, we postulate that the use of the Levy processes can generalise the use of the tree stochastic processes model (line 135). We justify our assumptions in line 137-155 (and Table 1).

Our assumptions are based on previous work such as (Williams 2003 and Williams 2004). Therefore, the main assumptions rely on the deviation from the mean for the non-stationarity of the noise, and homoscedasticity in regards to the sum of the two/three stochastic processes. Based on our simulations and experience with GNSS time series, the noise variance is finite (but very large in presence of tails). Note that we discuss about the finite variance in line 350-360 and appendix D.

(R): We will add the sentence “The main assumptions rely on the deviation from the mean for the non-stationarity of the noise, and homoscedasticity in regards to the sum of the two/three stochastic processes.”

- About the use of Levy Flights, the model may be limited for its application to GNSS time series due to the assumptions that GNSS time series have a finite variance. The randomness of the jumps in the GNSS time series is an assumption only valid for small offsets buried in the noise floor, because we cannot detect them by eyes and therefore difficult to model with a step function (See the discussion in appendix D).

- Your comment about looking at non-stationarity beyond the mean using

ARCH/GARCH type of models is interesting for future work (This will be mentioned in the revised conclusions). We have restrained the study on the common assumptions in geodesy with non-stationarity based on the variations of the mean value. It is also worth mentioning that when including a random-walk component, its amplitude is significantly smaller than the other two components (white and coloured noises) – see He et al. (2019).

- Intermittency in GNSS time series has also a limited application. Short high bursts or sudden large deviations for the mean are events which should be modelled (e.g. Earthquake and post-seismic relaxations), unless it is a fault/problem happening at the station. If such event is in the residual time series, it means that there is anxiety between the time series and the functional model. We discuss about this issue when justifying the use of the stable Levy process and the application of the Levy alpha stable distribution (See l. 216-217 and discussion in Section 3.2 – l.359-360).

- “You mention the use of the Hector software at several, in my opinion not necessarily relevant places. Since you do not provide any specific information that is unique to this software, I recommend mentioning this only in the acknowledgements and removing corresponding unnecessary statements elsewhere (e.g. ll. 200-201, 390, 402-403,417-418)”

The Hector software is mentioned and underlined in the manuscript as it was asked in the previous review. The interest of the software is that it is based on the joint estimation of the functional and stochastic models using maximum likelihood. The covariance can be modified to include different types of noise (e.g. White + power-law, White+ Flicker noise, ...). There are other software used in geodesy using other estimators (e.g. MCMC, Least-squares variance component estimator) which can give different results on the estimated geophysical signals due to different assumptions on the noise model and the stationary properties of the noise in order to limit the computing time (see Montillet and Bos, 2019). That is why as a geodesist, it is important to mention the software and to recall its specificity.

(R): we will limit mentioning the software too much in the revised manuscript.

- Âń Regarding the analysis of the modelling results, I am not convinced that it makes sense statistically to use correlations as a measure for the goodness-of-fit/matching between an empirical and a theoretical distribution. Âż

This approach was chosen in Montillet and Yu (2015). However, we will add the Anderson-Darling test as another measure.

- About the results in Table 3, we need to check them. The large standard deviation associated with the correlation means that there are a lot of variability in the data when applying the distribution. That can be due to the amplitude of the coloured noise with beta equal to 1 or 1.5.

Minor specific comments

Find below the answers to most of your questions from your bullet point list. Note that we will do another grammar check in the revised version.

- "I don't quite see that it is relevant to mention the length of the real-world series at this point"

The length of the time series is important, because it has been shown that it is very difficult to detect random-walk noise in short time series (e.g., L < 9 years). Also, He et al. (2019) showed that in some very long time series (e.g. 9 -10 years), the power spectrum can experience a flattening at high frequencies.

(R): We replace the sentence with : Note that the length of the geodetic time series (L) considered in this study is at least 9years (3285 observations) in order to be able to model correctly the coloured noise and to detect small amplitude random-walk component according to He et al. (2019).

- " Can you explain the relationship between slowly varying mean of colored noise and the Gauss-Markov assumption a bit more explicitly? "

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In L. 75 we recalled the Gauss-Markov assumption stating that the noise in GNSS daily position is Gaussian distributed. We assume that the coloured noise variations (around the mean) are slow and not big enough to be able to change the profile of a (multivariate) Gaussian distribution. Thus, that is related to intermittency and aggregation as previously discussed which could skew or completely deform the distribution with such large amplitude events.

(R): Let us add to the sentence "... we assume that the mean of the coloured noise is equal to μC(t), slowly varying with time, therefore ruling out the occurrence of specific events of large amplitude such as aggregations or burst of spikes which could invalidate the Gauss-Markov assumption".

- (R): About mentioning the Cauchy-class of processes. We will add in the paragraph (starting L. 90) the following sentence: " Another type of processes worth mentioning is the Cauchy-class of processes, which consist of the stationary Gaussian random processes defined by a correlation function which depends on the Hurst parameter which can be seen as the generalization of some stochastic models (Gneiting and Schlather, 2004). Âż

[Gneiting, T ; Schlather, M (2004) Stochastic Models That Separate Fractal Dimension and the Hurst Effect, 46(2), 269-282, SIAM Review, doi: 10.1137/S0036144501394387]

Note that the relationship between the fBm (Hurst parameter) and the FARIMA is recalled in L 437 in Appendix A.

- About the sentence L.154-155, it comes back to the previous discussion on the Gauss-Markov assumption. Also it is important to underline that in an ideal case where all the geophysical signals are well modelled in the time series, the residual time series should only contain the sum of the different noise components.

In order to create heavy tails distribution, it is most likely due to anxiety in the func-

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tional model by forgetting to model large offsets or post-seismic relaxation (e.g. when associated with slow slip events). Perhaps, it can be due also to unknown short-time transient processes. In a special case, it can be also due to the presence of outliers which have not been filtered. Therefore, the GM assumption may not be applied to the distribution of the residual time series.

(R): we will rewrite the sentence such as: "Therefore, the residual time series withholds some remaining unmodelled geophysical signals or unfiltered large outliers which can potentially undermine the Gauss-Markov assumption (e.g., presence of heavy tails in the distribution of the residual time series). Âż

- In L. 262, the driving parameters refer to the parameters of the characteristic function in Eq. 5. We will make it clear by replacing with "the parameters of the characteristic function".

- Ll.271-274: I understand this as that heavy tails in the series can either be attributed to the residuals or to the third component. Can such an attribution be actually unique?

We are not sure what the reviewer means. In order to have heavy tails, you must have large events unmodelled in the residual time series. In GNSS, it is unlikely that the various noise components produce large tails due to the amplitude of the noise.

- L.349: Doesn't the mentioned varying amplitude of the colored noise rather call for multiplicative/heteroscedastic models?

We value the idea of testing heteroscedastic models such as ARCH and GARCH for the future work. As previously said, this study is mainly based on a sum of various noise components. We will mention this future work in the conclusions.

- L.436-439: The link to fBm is a bit unclear here. Since you consider stationary processes (FARIMA class), it would be more reasonable in my opinion to link this to fractional Gaussian noise (fGn) the aggregation of which than provides sample paths for fBm.

(R): we will also mention the representation of the fBm via the fGn in the revised version.

Note that your comments on improving the readability of the manuscript and the better use of the Latex template will be taken into account in the revised manuscript. For example, the appendices will be added as supplementary electronic material.

Thank you for suggesting these improvements.

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