Comment on nhess-2021-119
Anonymous Referee #2

The paper by Kanai et al. 2022 'Robust uncertainty quantification of the volume of tsunami ionospheric holes for the 2011 Tohoku-Oki Earthquake: towards low-cost satellite-based tsunami warning systems' presents a new interesting statistical method to estimate the surface of the Tsunami Ionospheric Hole (TIH). The manuscript is well structured and described, and their method could help better understand the relation between the initial tsunami and the produced TIH. I recommend publishing the manuscript after considering the following general and minor comments.

General comments:

It makes sense to show the results of the surface fitting in 3D in Figure 5. However, the uncertainty (confidence interval) is not illustrative in 3D. I suggest showing the uncertainty estimates for the fitted surface in 2D instead of 3D, like the visualization in panels (g) and (h).

The authors claim that their surface fitting method reproduces the shape of the TIH in the same region, almost like the initial tsunami source by Saito et al. (2011b). The authors should better specify to what extent (area, wave height) the shape is reproduced. Do uplift or subsidence areas of the initial tsunami match the TIH? In how far are TECu and vertical displacement in meters comparable? Does the TIH mimic the initial tsunami shape? Here the authors could visualize better by comparing the initial tsunami source by Saito et al. (2011b) directly with their results. How does it compare to the initial tsunami source presented by other authors?

The authors should show a couple of other random cases with sparse data for the surface fitting. It will be visually beneficial for their work to demonstrate that their method could enhance future warning systems. Since other randomly chosen datasets should deliver similar results, the authors should demonstrate it in a figure.

TIH overlapping the initial tsunami:

The representation of the initial tsunami is problematic. It is not clear how the authors have chosen the sea-level threshold value to define the area of the initial tsunami. The
authors should represent the initial tsunami wave in m concerning the sea level of the event. Tsunamis may contain depression and elevation features in the wave field, which must be shown in the figure. There are many published source inversions for the 2011 Tohoku-Oki tsunami source, and the authors should compare the TIH to other published source inversions (e.g. Ammon et al. 2011, Wei et al. 2012, Yue and Lay 2013); otherwise, it seems they have chosen Saito et al. (2011b) that fits best to their results.

Moreover, it is essential to explore how the TIH overlaps the initial tsunami. The initial tsunami wavefield values should be compared directly to the TEC values.

On page 18, line 359, the authors claim that their method can estimate the tsunami region but the authors only use the Tohoku-Oki event. Before they can draw this conclusion, their method needs verification with other real cases. Moreover, it is not clear to the reader which TECu value (-2, -3) should be used to define the area of the initial tsunami. Is the same TECu value applicable for other tsunami cases?

**Minor comments:**

Page 3, line 56: Please define TECu the first time it appears in the text.

Page 5, line 132: Please define the variable O(n³).

Page 5, line 136: What is the difference between ‘more accurate, less uncertain and more robust’?

Page 6, caption figure 2: There is a space missing between ‘elementsto’

Page 6, line 148: The text could be deleted since the information is given in figure 3 'The red star is the location of the epicenter of the 2011 off the Pacific coast of Tohoku Earthquake and the two large black circles with slanting lines are outliers.'

Page 7, line 177: ‘which is shown using a red star mark,’ could be deleted since the information is in the figure caption.

Page 10, line 217: It is sufficient if the triangles' colour coding is in the figure 6 caption.

Page 10, line 219 – page 11, line 221: Can the authors explain why the tsunami source (Kamogawa et al., 2016) is relevant if they analyze the TIH expansion of their study? If they relate to the source in Kamogawa et al. 2016, they must show it in the figure.

Page 11, line 240: Any reference for the Hubeny’s distance formula?

Figure 7, Panel (a): Why does the TIH withdraw (05:59:30 - 06:01:30) in the southward direction before it starts to expand again?

Page 13, line 273: What do the authors mean? ‘if the TEC reduction is larger’. Larger than -2. Please give a more detailed and analytical comparison between the tsunami wavefield and the TEC field.

Page 13, line 277: What is meant by the ‘TIH almost overlaps the initial tsunami areas’

Page 15, line 295: Why does the volume of the TIH continue to increase until 28 minutes after the earthquake?

Page 15, line 297: Please quantify ‘huge simulated tsunami’ and ‘smaller simulated tsunami’
Page 16, Figure 9 caption: Please define acronym CI the first time used in the manuscript.

Page 16, line 302: Please define what is considered 'a huge' tsunami.

Page 16, line 317 to page 17, line 319: This information is redundant. The authors could delete the last phrase of this paragraph.

Page 17, line 327: ‘Also, the estimated TIH almost overlaps with the estimated initial tsunami area.’ As mentioned earlier, that statement needs better visual representation in figures. I also suggest further exploration, analyzes and discussion.

Page 17, line 342: Change the brackets from ‘Heki and Ping (2005)’ to ‘(Heki and Ping 2005)’. Please also correct the brackets in the conclusions or change the text accordingly for the references Zettergren et al. (2017), Zettergren and Snively (2019) & Shinagawa et al. (2013) on page 17, lines 345,347 and 348.

Page 18, line 365: The authors' comment that larger initial tsunamis cause larger decreases in TEC, according to Astafyeva et al. (2013) and Kamogawa et al. (2016): What is the relation between the size of the tsunami and the decreases in TEC. The authors use the volume of the TEC decrease as a measure for the TIH produced by the Tohoku-Oki tsunami. They should relate their measure to a corresponding measure of the tsunami size.

References:

