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Reply on RC2

Soohyun Yang et al.

Author comment on "Power-Law between the Apparent Drainage Density and the Pruning Area" by Soohyun Yang et al., Hydrol. Earth Syst. Sci. Discuss., https://doi.org/10.5194/hess-2022-237-AC2, 2022

We thank Referee 2 (R2) for helpful comments. Our replies to all comments are listed below. We will reflect any change addressed below into the revised manuscript which is requested in the subsequent step.

R2: The hypothesis behind the two derivations of the exponent η. Section 2.1.1. The Derivation 1 is developed in the hypothesis that $A_p = A_\omega$ (Page 5, line 123). However, eq (14) is derived in the hypothesis that $A_p << A_\Omega$. The authors should discuss on this assumption. Does it mean that the expression could be scale relevant and not appliable if ω is close to Ω? The same assumption ($A_p = A_\omega$) is done in Section 2.2.2 (Derivation 2), but then the equation (16) is generalized regardless the scale of the basin. Is equation (16) still valid if $A_p << A_\omega$?

Reply: In the Derivation 1, we intended the condition of $A_p << A_\Omega$to imply the limited boundary condition for a pruning area variable. Reconciling with the order-by-order A_p representation, it means that the power-law behavior in Eq. (13) with the exponent of Eq. (14) gets weaker as ω is close to Ω. Indeed, its behavior is caught as the finite size effect in the ρ_a-A_p relationship (see Fig. 2 in the original manuscript). To clarify it, we will add following sentences in Sec. 2.2.1 : "Note that $A_p << A_\Omega$ is an extreme expression intended for the neat derivation of the power-law relationship. Notwithstanding, it is also supported from the empirical boundary condition of to characterize fluvial channel networks (McNamara et al., 2006; Montgomery and Foufoula-Georgiou, 1993)."

In the Derivation 2, Eq. (16) is independent from the scale of the entire basin, because it is based on geometric representation of a river network. Please note that the treatment of $A_p = A_\omega$ is **not an assumption** but is merely to interprete A_p in the Horton-Strahler ordered distrete series. It is not sure what R2 meant by the condition of $A_p << A_\omega$because it would also depend on ω.

R2: It seems that the authors proposed a new expression (eq. 25) of η as function of the fractal dimension D_b just "observing the real river network" and finding a support of this choice considering that quarter-power scaling laws are widely found in biological systems. However, the proposed equation, even if it performs better in the four case studies as respect to the literature expressions, seems to always underestimate the η value. Why fixing an "a priori" coefficient? Even if the sample size is very low, why not deriving the coefficient, minimizing the error between observed and estimated values and proposing, as future

development, a testing of the new expression with a wider sample size?

Reply: Following R2's suggestion of "a testing of the new expression with a wider sample size," we have **doubled** the number of study networks from 4 to 8. On the basis of analysis of eight networks, our key results and conclusions remain the same. It is found that the seemingly under-estimation from 4 networks was merely coincidence. Out of total 8 networks, 4 networks exhibit η greater than observed values, while the rest exhibit smaller η. These results will be reflected in the revised manuscript.

With the increased number of networks, following R2's suggestion, we find the best-fit coefficient. Interestingly, the best-fit coincides with the proposed coefficient of 1/4. To clarify the estimation processes, we will add the following sentences near Eq. (25) in the revised manuscript: "Note that the coefficient of 1/4 is based on our analysis of real studied river networks (0.25±0.02)."

R2: Page 5, line 134. "… the overland flow length W…" Do the authors mean flow width?

Reply: We will clarify the terminology in Sec. 2.2.2 : "*W* is the mean overland flow length".

R2: Page 11, line 259-260. What does "satisfactory results" mean? Did they calculate any performance indicator? The sample seems quite small (4 basins). Can the analysis be considered robust?

Reply: The original paragraph with the term of "satisfactory results" will be discarded in the revised manuscript. In the scope of this study, we don't need to calculate a performance indicator. To present more robust arguments, we doubled study areas and conducted all relevant analyses. Further discussion on the updated results will be added in Sec. 4 of the revised manuscript as follows : "It is interesting that the simple Eq. (25) is well supported by analysis results, at least as good as the other two expressions. Theoretical derivations of Eqs. (20) and (23) rely on a fundamental assumption, i.e., Horton's laws hold precisely at all scales of a unit length to measure (La Barbera and Rosso, 1989; Rosso et al., 1991). Indeed, the assumption is too ideal to be satisfied for real river networks, as corroborated in the non-perfect straight fits when estimating Horton's ratios of our studied networks (Fig. S2 in SI). Moreover, the importance of fulfilling the assumption to employ Eq. (23) is demonstrated by Phillips (1993) studying very small catchments in the Southern Appalachians in the USA. This is the likely reason that Eqs. (21) and (24), derived on the basis of Eqs. (20) and (23), show greater deviations from the observed η values, than Eq. (25). As for Eq. (25), we suppose that incorporating the empirical approach into the theoretical D_b expression mitigates the likelihood of discrepancy between the estimated and the observed η values, compared to the other two expressions."