Comment on hess-2022-201
Anonymous Referee #2

Referee comment on "Frequency domain water table fluctuations reveal recharge in fractured aquifers depends on both intense and seasonal rainfall and unsaturated zone thickness" by Luca Guillaumot et al., Hydrol. Earth Syst. Sci. Discuss., https://doi.org/10.5194/hess-2022-201-RC2, 2022

Review of 'Frequency domain water table fluctuations reveal recharge in fractured aquifers depends on both intense and seasonal rainfall and unsaturated zone thickness’ by L. Guillaumot et al. HESS-2022-201.

Major comment

The paper analyzes the effect of recharge on groundwater level fluctuations in aquifer wells in the frequency domain through Fourier transforms of an analytical groundwater model. The authors invert the model to derive recharge fluctuations from water level fluctuations in wells, which is quite interesting. In the model calibration phase, the mathematical efficiency of the analytical model is taken full advantage of by carrying out a massive number of model runs that explore the entire parameter space on a regular grid.

The approach is original, interesting, and well-suited for HESS. At times, the explanation of the model and the mathematical techniques is a bit brief (see detailed comments). The method is applied to a pumped and an unpumped aquifer near the Atlantic coast of France, in a relatively humid climate.

From the fitting results in Fig. 6 it appears that the parameter identifiability would benefit from replacing the aquifer length as a fitting parameter by the characteristic time, but this is not discussed or explored in the paper.

A considerable weakness of the paper is that the model is calibrated for both aquifers, but
not validated, making it difficult to assess the model performance.

Overall, the line of thought is a bit hard to follow some times because individual sections are not as focused as the can be. Throughout the paper, the clarity can be improved by thinking about what exactly the authors wish to convey to the reader and how to do so clearly. In the detailed comments I indicate where I got completely lost. I hope this will lead to a more structured, coherent paper.

Sections 5, 6, and 7 are disappointing. They lack focus and structure and do not convey the main strengths of the study. That does not mean these strengths are not there. I very much like the modelling approach and the intended use. The Results and Discussion sections need to bring that out more strongly though.

**Detailed comments**

The title is informative but a bit long.

L. 18: Something is missing.

You use 'indeed' a few times, but it is unclear to me why.

L. 41: Potential recharge suggests this to be the maximum possible recharge in analogy to 'potential evapotranspiration', e.g., in the absence of evapotranspiration and overland flow. It would be equal to the rainfall rate, but with damping and delay due to the flow through the unsaturated zone. I am quite sure this is not what you mean. Perhaps use another term, like 'plausible' or 'probable'. From line 88 it appears 'instantaneous recharge' might also be adequate.

L. 75: ‘Although simple...’ Is not the simplicity of the model the reason it has the adaptability you highlight here?

Fig. 1: I do not understand ‘backward analytical inversion’. Is there a forward analytical inversion, as this term implies? If yes, what are the differences between the two, and are there any references you can provide?
The boundary conditions in the aquifer sketch do not seem to match those described in the caption.
L. 105: Both imposed heads are not allowed to vary in time, correct?

L. 120: This sentence is difficult to follow. Also, the wells are located in a 2D system (the map, if you will), while your solutions are one-dimensional. Is the well distance measured along the coordinate that is represented in the 1D system? If so, then you essentially spread out a point sink (the well) over the full width W. It would be a line sink in a 2D system then. This is not to criticize this approach, just to make sure I understand it correctly. If I am correct, than 'distance to the pumping wells' should be changed to reflect it is the distance along a single coordinate only, not a true Euclidian distance.

L. 121-125: Is that the reason you kept the boundary conditions time-invariant? Is this a reflection of the applicability of the superposition principle?

L. 157. Apparently, I misunderstood you at L. 120. Is the aquifer length then related to the half-distances between pumping wells in the aquifer? This needs to be explained better.

L. 158: In a similar fashion, is the aquifer length related to half-distance between adjacent rivers?

N.B. Reading on, I see that you discuss this in section 3.2. Perhaps give some of that info here and refer for details to section 3.2. I do not find your choice for a constant head BC terribly convincing. Is it mathematically or physically necessary? If so, please point that out more clearly.

L. 157 and 158: It appears to me that if you base aquifer lengths on distances between rivers and wells, the boundary that is not the river or the well should be a no-flow boundary, not a prescribed head boundary.

L. 164-165 (and possible elsewhere): Units are usually not in italics, and the multiplication dot is unusual.

L. 177: ‘...along time.’ I do not understand.

L. 182: The double division signs of \(Q/W/L\) are confusing. I presume you want to divide \(Q\) by the aquifer area.
L. 183-185: Unclear sentence, please rephrase.
From this sentence it is not clear how anomalies are defined.

The dependency of $C_{xy}$ on $\omega$ is missing. One of the $P_{xx}$ should be $P_{yy}$.

Section 2.6 is so brief it is difficult to follow for me, as a reader who is not familiar with this technique. Please expand and add some references that give the basics of the methodology.

Panel c has wells that are not mapped on the cross-section depicted in panel b. The dashed blue line in pane c is the phreatic level, right? Please mention this in the caption or the legend.

How does this observation (high sensitivity of deep fractures to recharge) affect the nature and the role of the unsaturated zone model for modifying the recharge signal?

Why does the SURFEX recharge appear to increase and then fall before heavy rainfall?

You only show the results of the calibration. A validation step is missing. This is a major weakness of the paper.

'L/T appears … in equation 2'. So does $S/T$, through the characteristic time. I am not sure this argument is valid.

The characteristic time is fully defined once the storage coefficient, transmissivity, and aquifer length are determined. How can it then be a fitting parameter along with the other three? (Earlier in the paper, it was not.) From Fig. 6 it would appear a good idea to use the characteristic time instead of the aquifer length as a fitting parameter and determine $L$ from the other three.

This paragraph starts clear, but then you lost me. What does 'inter-compared' mean, for instance? And how can you reduce noise amplification during backward modelling by doing something that apparently makes graphs easier to read?
L. 363: How does Fig. 8 show similarity between wells? Also, the ‘noise’ can be easily explained physically. Why is it called noise instead of fluctuations or temporal variations caused by pumping?

L. 372-375: I can find neither a graph nor a table backing up these statements.

L. 380: Thornthwaite does not show anything in winter 2002.

L. 383 and 387: What are rainfall and P-PET efficiency?

Section 5.3 uses terminology with which I am too unfamiliar to understand what points are being made. Throughout the paper, the English is a bit off, but here it somehow becomes so much so that I can no longer decipher the meaning of several sentences. This section needs to be thoroughly rewritten to be accessible to readers outside the immediate field of this paper, and the English needs substantial improvement.

Sections 6 and 7 require over five pages of text. That is rather long. They also introduce a large number of new references, which indicates that the paper is not well organized. The Introduction and the Methods sections should cover most of the literature needed in the paper.

L. 499-502. What is the point of this paragraph?

Section 6.2.1. This is some kind of recap, mixed with a discussion. I like the evaluation of the soil models but do not really know what to do with the rest of this text. It reads like a brainstorm session.

Section 6.2.2. What is ‘linear behavior’ in this context and how does it link different phenomena? Is a ‘linear coefficient’ (which normally is a single-valued number, not a curve), in reality the slope of a linear relationship that you have at this point not properly defined? This is an example where lax formulations obscure what I believe to be a useful message.

L. 530: I know of very few examples, other than interflow, of lateral flow in the unsaturated zone.
Section 6.3.1. This does not really arise from the Results, does it? Also, it is a bit obvious.

L. 534: This info can be moved to the site description earlier in the paper.

Section 7: Only the last two paragraphs are conclusions, the rest is more of a summary.

Appendix A. You do not present a formal Fourier transform of the governing equation and the boundary conditions, which would perhaps make it easier to understand the line of thought and you choice to make the prescribed heads, but not the prescribed flux, time-invariant.

I can follow most of the development, but nevertheless would like to have more references and a few more steps in the derivations. I am not intimately familiar with the techniques you use.

You never clarify how many terms of the infinite sums are required to achieve convergence, or what criterion you used to define convergence.

L. 585: ‘...of the aquifer’ There is a symbol I do not know after ‘aquifer’. Typo?

Eq. (A2): The only time-dependent variables appear to be R(t) and h(x,t). Is that correct?

L. 593: The steady-state part of Eq. (A1) is that the second derivative of h with respect to x equals a constant recharge rate, correct? In that case, should C3 equal the mean recharge rate?

L. 600: Mixed-type (Cauchy BC) are not permitted?

Eq. (A9): Are the first three terms the parabolic groundwater level found for steady flow towards drains modeled as fully penetrating ditches?

It is not clear to me how you arrive at Eqs. (A10) and (A11).
L. 630-633: This is not a real test, is it? You modeled numerically the simplified set-up that permitted the analytical solution. This used to be done when in the 1970s and 1980s to test the accuracy of numerical models but was never intended to evaluate the analytical models.

Supporting information: If you have a radial aquifer which is pumped in the middle, does that not lead to a singularity at the center of the radius of the well is zero?