

Geosci. Model Dev. Discuss., referee comment RC2 https://doi.org/10.5194/gmd-2022-69-RC2, 2022 © Author(s) 2022. This work is distributed under the Creative Commons Attribution 4.0 License.

## Comment on gmd-2022-69

Anonymous Referee #2

Referee comment on "A local particle filter and its Gaussian mixture extension implemented with minor modifications to the LETKF" by Shunji Kotsuki et al., Geosci. Model Dev. Discuss., https://doi.org/10.5194/gmd-2022-69-RC2, 2022

## General comments:

The authors present a useful incremental step in developing a practical particle filter based methodology that can be scaled to dimensions required for realistic operational forecasting. The results show a narrow range of parameters for the augmented LPF (i.e. the LPFGM) that can outperform a tuned LETKF implementation using a low-resolution global primitive equations model.

I think the manuscript would be improved if the authors could highlight a specific example case/scenario in which the dynamics require a non-Gaussian method. At its simplest, this could simple be to focus in on a regional assessment of the Southern Hemisphere East Pacific Ocean RAOB case (or the Arctic, using stenographic project map), where large differences were found between LETKF and LPFGM. It would be useful to see RMSE and ensemble spread statistics calculated for this region alone, and perhaps a bit more detailed assessment of this region as a 'case study'. This could provide further motivation for finding cases in which the LPFGM enhancement to the LETKF software architecture could provide significant value.

More strength could be given to the argument for using this method if it could be made clear that such scenarios are common occurrences in operational systems (or perhaps in reanalysis systems that focus on less observed periods in history). Are there any scenarios for a coupled Earth system forecasting system in which particular regions or physical quantities are particularly poorly observed that might benefit from this method? Do the authors see applications in reanalysis such as CERA-20C and the NOAA 20th Century Reanalysis that might improve historical reconstructions during relatively sparsely observed periods? Are there any applications in the modern satellite era where this approach could still be advantageous?

Specific com	nments:
L 19:	
"Therefore,	implementing [the LPF] consistently with an existing LETKF code is useful."
L 21:	
"This study	develop[s]"
L 25:	

" The LPFGM showed more accurate and stable performances than the LPF with"
Change to:
" The LPFGM showed more accurate and stable performance than the LPF with"
L 29:
"The SPEEDY-based LETKF, LPF, and LPFGM systems [are] available"
L 37:
L J/.

"Ensemble[-based] data assimilation (DA)"
L 37:
"such as weather and ocean predictions."
Change to:
"such as weather and ocean prediction."
L 44:

"models shows local low dimensionality "
Change to:
"models show local low dimensionality "
L 44:
L 44.
"and practical EnKF with these systems uses"
and practical link with these systems uses
Change to:
"and practical EnKF implementations use"

L 45:
"that limits the impact of distant observations within a local area and reduces the effective degrees of"
Change to:
"that limits the impact of distant observations while also reducing the effective degrees of"
L 52:
"issues in [the basic assumptions of the] EnKF [by permitting] nonlinear observation operator[s] and"

L 57:
"particles or the ensemble size [must] be increased"
L 60:
"equivalent-weights particle filter (ETPF"
Is this the correct acronym? It seems that EWPF would be more appropriate.

L 64:
"method [for the] EnKF"
L 76-77:
", and the source code is accessible at https://github.com/takemasa-miyoshi/letkf."
It seems to me that this belongs in the data availability section, or something similar, toward the end of the document.
L 100:
It looks a bit odd to 'divide' the matrix by beta - I think it would be more clear to write $((m-1)/beta)$ before the matrix "I".

L 103, equation (5):
The second equality of equation (5) is perhaps not obvious. It would be helpful to add a step or two showing this relationship. (For example, showing how the relationship can be derived from the first equality in eqn. 5 and the definition in equation 6.)
It seems a bit odd the way it is written here because Pa is defined using K.
L 123:
"and relaxation to prior"
Do you use "relaxation to prior spread", or "relaxation to prior perturbations"?

I would suggest:
"and a relaxation to prior scheme (Zhang et al., 2004) as implemented by Whitaker and Hamill (2012)"
L 145:
It might be worth mentioning that this implies that the observation error distribution is assumed Gaussian.
L 146, equation 12:
It looks like the "T" transpose operator is bolded.

L 167, equation 19:
Could you provide some explanation of condition #1. The "m" isn't defined, nor are the indices i,j explained. The meaning of the superscripts is not explained.
L 185:
" In addition, this stochastic approach approximates Eq. (19) using the Monte-Carlo approach."
Nice approach.
L 185-186:

"The generated transform matrix with 200 samples (Fig. 1 c) is close to that with 10,000 samples (Fig. 1 d) in the case of 40 particles. $^{\prime\prime}$
I'm curious how this Monte Carlo approach scales with the ensemble size. How does the performance (e.g. accuracy/stability of the method and the computational costs) change as the ensemble size gets large?
L 189:
"The effective particle size N_eff"
This terminology is a bit confusing, as a particle is analogous to an ensemble member. I would either say "the effective ensemble size" or "the effective number of particles".
L 195, equation 23:

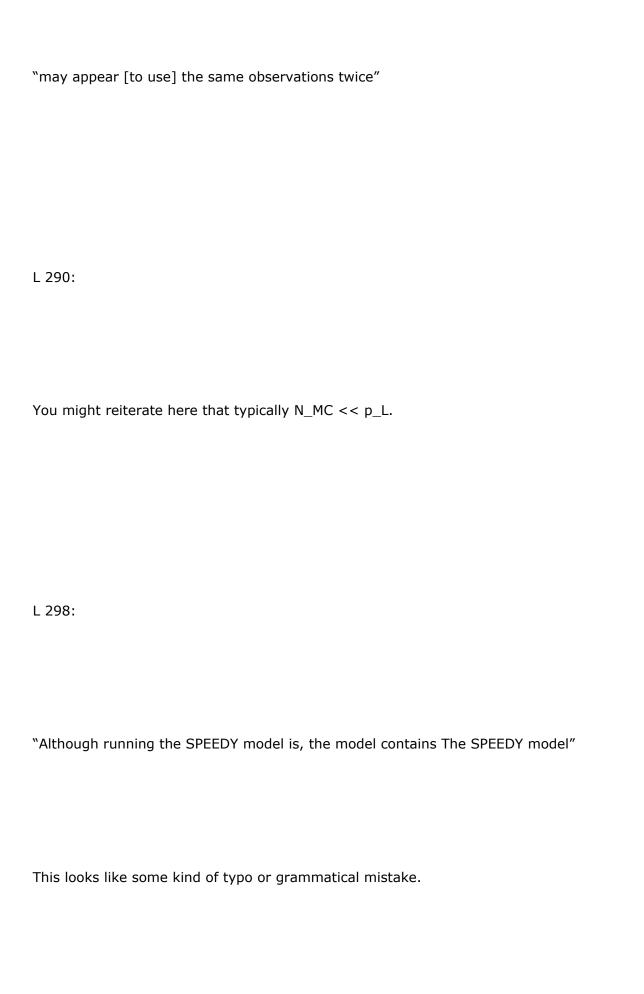
Is this effectively a relaxation back to equal weights? I think it would be helpful to explain this more, especially since later it is indicated that only values of tau as 0 and 1 are used. What then is the implication of these two choices?
L 203:
"Therefore, the LPF usually applies inflation to the posterior particles (e.g., Farchi and Bocquet 2018)."
Change to:
"Therefore, the LPF usually applies inflation to the posterior particles (e.g., Penny and Miyoshi, 2016; Farchi and Bocquet 2018)."
Penny and Miyoshi (2016) also applied a form of additive inflation to the posterior:

"at each cycle we add Gaussian noise with variance scaled locally to a magnitude matching the analysis error variance and apply this to each analysis member prior to the subsequent ensemble forecast. The amplitude of the additive noise was chosen to conform to the dynamics of the growing error subspace, as estimated by the analysis ensemble spread." (Penny and Miyoshi, 2016)
L 206:
"based on [the] authors' preliminary experiments"
L 214:

"The Gaussian mixture extension of the LPF is [one] such hybrid algorithm"
L 218:
"hut"
Change to:
"hat"
L 223:
How does increasing gamma reduce the amplitude? Is there a relationship between gamma and the number of ensemble members?
L 226, equation 26:

Am I interpreting correctly that the Kalman filter is applied to each participle independently, using the forecast error covariance estimated from the entire ensemble? Or, is it applied to the adjust the mean of the individual particles like an ETKF?
L 228:
I see that $K^{\wedge}$ is defined here, but I do not see a definition for Pa $^{\wedge}$ . It seems that Pb $^{\wedge}$ is a scaled version of the ensemble forecast error covariance, but I'd like to see a more precise discussion of how $K^{\wedge}$ is formed - is it the same for every particle, or does each particle have a different $K^{\wedge}$ ?
If all of the particles have the same K^, then how is this different than applying the standard EnKF update to the mean and perturbations? Please provide more discussion on these points.
L 235:

Do you mean T_{t,GM}?
L 241:
What is WP22? Can you just provide the full reference?
L 247, equation 33:
In practice, would there be any benefit to forming and applying the $T_{t,LPFGM}$ matrix directly?
L 249:



L 307:	
"Gaussian noises were added"	
Change to:	
"Gaussian noise was added"	
L 308:	
"data whose interval is 6 h"	
Change to:	

"data at 6 h intervals"
L 311:
"every seven layers"
Change to:
"all seven layers"
L 311-312:
"The ensemble size is 40 and their initial conditions were taken from an independent nature run."

This is not clear - was a free-running ensemble used as the nature run? Or was the initial ensemble sampled from a single deterministic nature run? Please be more precise.
L 320:
" LETKF were optimized manually"
Change to:
" LETKF were tuned manually"
If this was done manually, then it is likely not optimized.

L 417-418:
"An alternative method of inflation is adding random noise to the transform matrix (Potthast et al., 2019). However, regulating the amplitude of the random noise was not trivial in the authors' preliminary experiments with L96 (not shown)."
Have the authors tried inflation method as detailed by Penny and Miyoshi (2016). There
the amplitude was specified by the local analysis error variance, and could be interpreted as noise applied to the transform matrix (it is applied to the analysis perturbations).
L 446:
"This study considered two experimental settings with and without weight succession"
It seems like at least one case in between (e.g. $tau = 0.5$ ) should be tried to give some idea of the benefit of this weighting method.

L 456:	
"optimal localization scale "	
Again, if not mathematically optimized, then I would change this to:	
"best performing localization scale"	
L 461:	
"forecast in a sparsely observed regions"	
Change to:	

"forecast in sparsely observed regions"
L 700, figure 9:
Please use white instead of yellow for the "near zero" values in panels (d) and (e).
(Same comment for Figure 12)