

Geosci. Model Dev. Discuss., referee comment RC1  
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## Comment on gmd-2022-225

Anonymous Referee #1

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Referee comment on "A mixed finite element discretisation of the shallow water equations" by James Kent et al., Geosci. Model Dev. Discuss.,  
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### Summary

The article presents a new numerical method for solving the shallow water equations on the sphere that employs mimetic finite element methods on a cubed-sphere grid. It represents a significant advance from previous work, extending the methods to spherical geometry as a step toward development of an accurate and computationally efficient atmospheric dynamical core for operational use.

As a replacement for an operational model that uses a latitude-longitude grid, the article correctly suggests that this method is capable of significant improvements in parallel scalability on the latest computing architectures.

Results from the standard test cases are presented, and demonstrate that the model performs appropriately.

Advanced numerical techniques such as mimetic finite element methods preserve significant properties of the continuous equations in their discrete form and are capable of achieving high efficiency in the massively parallel simulations required by high resolution atmospheric models.

The article represents an important step toward development of such a method, and I recommend it for publication with minor revisions, as suggested below.

### General comments

While mimetic methods are becoming more common, it seems a lot to ask of the interdisciplinary GMD audience to follow a discussion of function spaces and de Rham complexes without some assistance. An illustration such as Figure 4 from Melvin et al. (2018) that corresponds to the specific cases mentioned by equations (7) and (8) would be most helpful.

Similarly, the article could benefit from a quick reminder of why mixed finite element methods are useful, especially since the lowest order formulation is used here. Differences between finite element methods, finite volume methods, and finite difference methods often disappear when used with low order discretizations. What is gained here that would not be present in a staggered finite volume method such as the one presented by Thuburn et al. (2014)?

Given the emphasis given to computational efficiency in the introduction, I expected more discussion of the method's computational performance. Detailed scaling studies are likely unrealistic this stage of development, but some general discussion would be helpful. Are the expected gains strictly due to the choice of grid, i.e., cubed sphere vs. ~latitude-longitude? Or is the numerical method helpful, too, for example, are its stencils for field reconstruction (e.g., Figure 1) smaller than other methods, implying less communication is required during runtime?

### Specific comments

- Equations (5) and (6) suggest that the advecting velocity is  $\overline{\vec{u}}^{1/2}$  even in cases where  $\alpha \neq 1/2$ ; is this true? Wood et al. (2014) suggest that off-centering by setting  $\alpha > 1/2$  is important in the context of a 3D deep atmosphere model with orography. Is that concern relevant here, given that the method is presented as a step toward a full 3d atmosphere model?
- How many iterations of GMRES are typically required to solve (33)? How sensitive is this number to the resolution?
- The description in Section 6 of a "finite-element representation of the sphere within a cell with polynomials" is difficult to follow. I assume that the four vertices of an element lie on the sphere; for the case of a quadratic elements, are the nodes that are not vertices also on the sphere?
- Is the fact that some internal points of a cell may not exactly lie on the spherical surface related to the fact that different function spaces are used for different variables? It doesn't seem to be an issue with other finite-element dynamical cores such as Guba et al. (2014), that also rely on mappings to and from a reference quadrilateral.
- I found the discussion of error at the beginning of Section 7.1 confusing; it states that the method is second-order in both space and time, but immediately preceding this remark at the end of Section 6, fourth-order convergence is cited as the reason for choosing quadratic elements. I agree that the method should be overall second order, so I presume that the fourth-order convergence refers to reconstructing the spherical surface itself, rather than an arbitrary scalar function?

### References

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