

Geosci. Model Dev. Discuss., referee comment RC2  
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## Comment on gmd-2021-296

Anonymous Referee #2

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Referee comment on "Islet: interpolation semi-Lagrangian element-based transport" by Andrew M. Bradley et al., Geosci. Model Dev. Discuss.,  
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This manuscript describes an interpolation-based semi-Lagrangian (SL) method for the transport problem on spectral-element (SE) domains. The SL transport schemes are widely used for multi-tracer transport in atmospheric models due to their accuracy and computational efficiency. The classical SL method employs interpolation at the upstream locations of the backward trajectories to estimate the advecting scalar values at the new time level. However, such an approach is not conservative per se, for practical applications an arbitrary procedure known as the "mass-fixing" usually employed for global conservation — which may have an adverse effect for climate-scale (long term) integration due to the local mass drifting. On the other hand, a finite-volume formulation of the SL method is conservative by design, where the upstream interpolation over the Lagrangian element is replaced by integration constrained to be locally (hence globally) mass conservative. The conservative data transfer from regular Eulerian grid to the deformed Lagrangian grid often referred to as the remapping (re-zoning), a limiter or shape-preserving scheme is usually employed for physically realizable solutions. A wide body of literature is available for both conservative and classical SL methods.

Implementation of conservative SL method on spherical domains tiled with high-order spectral-elements are very challenging. Authors have proposed an interpolation-based SL method *Islet* for the SE discretization. Instead of using the "unstable" native high-order interpolator (basis function) they have devised a cumbersome numerical procedure which employs an alternative grid system within each spectral element, adding another layer of complexity. The Islet method is not conservative, nevertheless, the global conservation is achieved by mass-fixing. The authors argue that the Islet method can handle tracer transport as well as the remapping between *physics* & *dynamics* grids, and incorporate shape-preservation filters.

The manuscript is very long, the Islet interpolation as described by the authors is extremely complex. Authors failed to explain the core interpolation algorithm with clarity, there are many statements in the manuscript which leads to ambiguity. The numerical analysis part is very intense maybe more suitable for a computational math journal (e.g., SIAM / JCP) than the GMD. The subject covered could be split into a two-part paper, one describing the basic algorithm and analysis with more details and rigor, and the second part for implementation and validation with standard tests. This would be helpful for better reading. The current manuscript is written in an awkward manner and is unacceptable for publication.

**Recommendation:** Major revision, possibly resubmit as a two-part manuscript. Authors should address the following questions.

(1) The stability associated with the SL method is that the deformational Courant number (Lipschitz condition) should not exceed unity, in plain language, the trajectories should not cross intersect (see, Staniforth & Cote's 1992 MWR paper). Is the cubic ISL method (lines 115-120) unstable due to this condition? Need some explanation.

(2) The SL transport scheme can be stabilized using a limiter, filter or with an explicit diffusion (see, Ullrich & Norman, QJRM, 2014). You can use the native high-order SE interpolation (basis function) for the SL transport combined with the limiter which you are already using for the Islet method. It will be interesting to see how the Islet method compares with this simple SL-SE scheme employing 4x4 GLL grid (I guess that is the SE grid choice made for the operational E3SM).

(3) It is not convincing to have 3 grid systems (physics: FV, dynamics: GLL, transport: tweaked GLL) in a SE modeling framework. The Fig.5 shows such a grid configuration, and it appears to be very challenging. At a very high (NH) resolution the data

movement is a major issue for an element-based Galerkin model (DG/SE). A typical climate model may have  $O(100)$  tracers, an additional tracer grid with more DOF than the dynamic grid can exacerbate this problem. This will limit the use of Islet scheme, how do you address it?

(4) With real data you have velocity information only available at the GLL (dynamics) grid, the way you find the 2D trajectory information using the 3D Cartesian coordinates leads to additional computational overhead when the method is extended to the 3D application (line 470-475). This needs some justification, why not use the spherical  $(u,v)$  components or corresponding contravariant vectors? It is not clear that the maximum eigenvalue required for the interpolation is the tracer data dependent, in that case you have a serious computational overhead for the multi-tracer applications, Please clarify! What is the computational halo requirement for an SE stencil with  $N \times N$  GLL points, when the shape preserving limiter is applied?

(5) What is the special advantage of using Islet method? It seems you have introduced a complex numerical method for a relatively simple linear transport problem. If mass-fixing is the way to go, one could use the RBF-based (Kriging type) interpolator which provides very accurate solution, and no need for the expensive search for max eigenvalue etc.

(6) The results are looking good, authors should limit the number of figures and make an effort to compare the results with that of other high-order element-based schemes. Why the results from your own previous papers (Bosler et al. 2019, SIAM J Sci. Computing; Guba et al. 2014, JCP) discussed? These results should be compared and the relative merits should be discussed.

(7) There are many undefined terms (e.g. CAAS) and notations which I am going to list, this should be fixed.