Responses to referee 1:

The manuscript ‘Quantifying Causal Contributions in Earth Systems by Normalized Information Flow’ by Cheng and Redfern addresses an important scientific question: how do we best measure causal relationships in coupled systems. While a good part of the manuscript is well-written and while I generally find the topic exciting and worth publicising, I found it impossible to assess for myself if the presented method is indeed the advance it is claimed to be. This might be a result of me being unfamiliar with the cited Liang papers. In any case, I therefore stopped with a detailed review of the paper after section 3.1, because I felt I was unable to follow the detailed arguments and maths purely based on the material presented, and I unfortunately lack the time to read up on all of the cited literature myself. I thus conclude that a major revision to the introduction of the method is essential (at least) before I could recommend publication. I also feel that a clearer and more intuitive discussion of the results and examples would be a massive step forward in terms of increasing the potential impact of this paper. Below, I list a few detailed comments that I hope the authors will find helpful.

We will revise the manuscript to update new references and improve its readability. In particular, we will incorporate the examples shown above which clarify the concerns raised here.

Major points:

- I indeed think that the comparison to other work could be extended (see also other comments in the public discussion). Next to the references included, this should also involve the following citation: Runge et al. Detecting and quantifying causal associations in large nonlinear time series datasets. Science Advances 5, eaau4996 (2019). https://advances.sciencemag.org/content/5/11/eaau4996.abstract

We will revise our introduction and discussion with further updated references.

- page 3: the motivation of the IF and nIF formulas (2) to (4) needs to be clarified. What is the intuition behind them? Given how central it is to the paper, a reference to Liang 2014 is not sufficient. In this context, a comparison to other methods might be
particularly insightful. A discussion, which may well be focused on the ‘big picture’, would certainly also be useful for the general reader who may be less accustomed to think about concepts such as ‘transfer entropy’ and ‘causality’. In particular, a better explanation of why the method is different from a simple linear correlation (which can be causal or not) is required. Overall, I suggest a significant extension to section 2.1 (a page or more, stepping through the equations; describing them intuitively, maybe even using examples for each variable).

Our revision discusses the limitations of regression method and the most commonly used causal analysis (Granger causality). We hope the new key finding mentioned in the beginning of these responses help address the concerns. Nevertheless, detailed comparison with all other causal analysis is beyond the scope of this work.

Further corrections:

- page 1, L.14/15: do you maybe mean: …, ‘especially concerning a network of teleconnections’?
Yes

- page 2: the first two sentences are duplicates. Please revise.
Noted. Thanks.

- page 2 l. 23: would downtune to: ‘yet established’
Agree. Thank you.

- page 2 l.28 you mean: ‘good quantitative measures of causal strength’? How would you define good here? Would be careful/thoughtful about the wording.
Noted. We will rewrite the sentence. Nevertheless, since this manuscript focuses more on the practicality of methods based on mock-up data verification, rather than derivation of the fundamental theory behind our hypothesis, it may be difficult to avoid all subjective adjectives in some places.

- page 3. L. 18/19: I don’t understand the reasoning for using \( m \text{abs}(nIF) \) here. Could you clarify?
Initially, we suspected that \(|nIF|\) plays similar role to \( R^2 \), and we hence replaced \( R^2 \) in \( mR^2 \) by \(|nIF|\) to tested the outcome. This is, however, proved rather unnecessary since it does not resolve the problem of \( m \) from regression method with low \( R^2 \). We intend to remove discussion of \( m|nIF| \) as a quantifier in the revised manuscript to be replaced by the \( md.nIF \), as discussed at the beginning of these responses.

- do you think only a linear and second order comparison is sufficient to establish a benchmark? How about other, e.g. non-parametric forms, which can capture more complex non-linear relationships?
We are not trying to establish a benchmark that \(|nIF|\) (or \( md.nIF \)) would be the best approach for estimating causal contributions for Earth system. There are many useful modelling approaches, including artificial intelligence etc. Nevertheless, we have established and empirically verified a simple and useful equation with hypothesis that the \( md.nIF \) is proportional to the causal sensitivity. To address this concern, we suggest that
we may rephrase our title to “Assessing the Practicality of (modified) Normalized Information Flow for Quantifying Causal Contributions in Earth Systems”.

- **Can you clarify why a calibration factor might be needed in reality? What does that tell us about the correctness/suitability of the approach?**

As highlighted in comment AC2, the calibration factor is approximately equal to the maximal \(|m|\) when \(R^2 \rightarrow 1\) for \(md.nIF\), or \(2 \times\) maximal \(|m|\) when \(R^2 \nrightarrow 1\) for \(nIF\). Our simple example illustrated in Figure R1 (of AC2) also illustrates that the strength of causality (the proportional relationship) is better reflected by \(IF, nIF\), and \(md.nIF\) as compared to \(m\) in regressions.

- **page 4, l. 2/3: isn’t the second term an important self-feedback aspect? I think this is quite a broad approximation that would require a problem-specific justification.**

Indeed, there is uncertainty associated with a series of variable arising from a cause variable, including self-dependency, and impacts from other cause variables amongst other factors. A feedback loop is complete only when causal contributions from both directions are present. Therefore, we now focus on \(IF(X \rightarrow Y)\) and \(IF(Y \rightarrow X)\) and their (modified) normalized forms. As discussed at AC2, there remain issues with the two terms in the normalized information and that will require further work beyond the scope of the current manuscript.

- **page 4, l. 9: at this point it is still unclear to me why the magnitude would imply a strength of causality, or wrt l.5 why IF represents a ‘trend of uncertainty’. This has to be introduced more carefully and intuitively. I am sure that should be possible and is key to improve the accessibility of this paper.**

At line 5 we discuss the original sign of \(IF\). In brief, we consider that the “flow of uncertainty (i.e. magnitude of \(IF\))” represents “causality”. When \(IF\) is positive, our interpretations is that the cause variable is providing more “uncertainty” to the effect variable, or destabilizing the effect variable. Negative \(IF\) implies reduced uncertainty received by the effect variable, or stabilization of effect variable. Note that this explanation may not be the final word in the significance of the sign of \(IF\), especially when the inventor Liang now tends to promote the use of its absolute magnitude. We will clarify this point in the revision, but shall not visit it in depth since the sign of \(IF\) is replaced by the sign of correlation which better describes whether the two variables are amplifying or attenuating each other.

- **31: in such a case**

  Noted.

- **32: and how about the corresponding feedback of methane ON temperature?**

In the revised manuscript, we will cite Liang’s more recent paper which highlights that indirect causal influence is usually with much lower \(IF\) than those with direct influence. This is why it can still be applied in cases with bidirectional feedback. In our first version of manuscript, all mock-up data examples contain bidirectional causal contributions, and the method still reasonably reflects the direct causal contributions. From physical principles, we already know that additional methane warms the climate, but that is not the focus of this paper and does not lessen the value of applying our method to understand how climate affects atmospheric methane concentrations.

- **page 5: the entire discussion of the cases in Figure 1 and the test case should be extended and could be written much more clearly. I really have difficulties to follow.**
suggest you motivate each problem and give real examples (even if you don’t calculate them – maybe point to citations). In Figure 1- are X and Y multidimensional in most cases I assume? If yes, this should be visualized as well compared to case (a).

Figure 1 illustrates the complexity of real-world problems with multiple causes and teleconnections. Examples given include the mock-up data and the methane-climate feedbacks. We will explore how to improve the clarity.

- Figures 2, 3 and 4 are hard to decipher in terms of size of the labels.

Noted. We will explore how to improve the Figure readability.

Citation: https://doi.org/10.5194/gmd-2021-196-RC1