Reply on CC1
Chin-Hsien Cheng and Simon A. T. Redfern

Author comment on "Quantifying Causal Contributions in Earth Systems by Normalized Information Flow" by Chin-Hsien Cheng and Simon A. T. Redfern, Geosci. Model Dev. Discuss., https://doi.org/10.5194/gmd-2021-196-AC1, 2021

We are grateful for these comments, especially for sharing the recent papers by Liang. Where appropriate, we will incorporate these into our revised paper and reframe our discussion wherever appropriate. In general, the findings from Liang's recent papers are consistent with ours and help to better contextualise some of our findings. In particular, we will certainly cite the following:

- Liang, X.S., *Normalized Multivariate Time Series Causality Analysis and Causal Graph Reconstruction* Entropy, 2021. \textbf{23}(6): p. 679. The success of information flow under heavy-noise conditions demonstrated here supports our findings, showing the potentially improved estimates of causal contribution estimate given by IF. We are also grateful for you pointing us to "X. S. Liang, F. Xu, and Y. Rong, El Niño Modoki thus far can be mostly predicted more than 10 years ahead of time. Scientific Report, under revision" which we will be happy to cite.

- The first paper above, Liang (2021) Entropy 23: 679, also shows the capability of IF and nIF in analysing causality among multivariate time series and identifying cofounder and self loops. In another paper (Liang, X. *Measuring the importance of individual units in producing the collective behavior of a complex network*. 2021. arXiv:2104.09290) it is noted that “the node with largest information flow is indeed most crucial for the (causal) network”. For our empirical examples, the causal contributions are calculated based on the interdependent function while the dependencies carried forward over self loops are not counted as causal contributions. The character of IF shown in Liang’s papers explains why these carry forward dependencies do not significantly affect the estimates.

- We have pointed out the major inaccuracies for estimating causal contributions by nIF under large noise conditions coming from the noise instead of IF. As a result, a potential future improvement in estimates of causal contributions might be achieved by differentiating the self-dependency information flow from other noise and considering more than one parent variable. We have noted this difficulty since “cumulative information flow does not equal to the sum of the information flows to other individual units”, as pointed out by Liang in *Measuring the importance of individual units in producing the collective behavior of a complex network*. 2021. arXiv:2104.09290.

Regarding your second comment: “the authors suggest using Pearson correlation sign to correct IF causality… seems to ignore the differences between the meaning of the signs
of $IF$ and the meanings of the sign of Pearson correlation (PC); this misconception may be a result of insufficient clarification in our manuscript, which we can address. We would like to acknowledge the difference between the signs determined by these two methods. Both can be useful but neither of them can replace the other. From the application point of view, in estimating the positive vs negative feedback between two time-series, the sign of correlation or covariance is more directly relevant. In our empirical assessment, we only try to verify our hypothesis (equation 7) and whether integrating the “magnitude of (normalized) information flow” and the “sign of Pearson correlation” could be useful. We already point out that indirect qualitative application of $IF$ or $nIF$ does not always fully utilize the determined causality, especially if we wish to quantify the varying interdependent contributions between causally related variables. We therefore try to make better use of the determined magnitude of $IF$ and $nIF$, while agreeing that the application of the sign of $IF$ should be explored further.