

Geosci. Model Dev. Discuss., referee comment RC2  
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## Comment on gmd-2021-122

Anonymous Referee #2

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Referee comment on "UBER v1.0: a universal kinetic equation solver for radiation belts"  
by Liheng Zheng et al., Geosci. Model Dev. Discuss.,  
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This paper presents a very nice universal solver for transport equations with examples in various situations, utilizing the SDE method. It is always happy to see that significant improvements have been made to the SDE method in terms of speed and handling of boundary conditions, since its first adoption to the radiation belt problem in 2008. The SDE method clearly has its advantage in terms of handling cross-diffusion terms and efficiency the case when only a few solutions in  $(t,x)$  are needed. In principle, I think this paper is publishable. I only have a few minor comments and suggestions for the authors.

1, The main problem I have with the paper is that it failed to discuss the main concern of the the SDE method: its speed, especially compared with finite difference methods etc. Clearly, the SDE code with newer techniques is much much more efficient than the traditional one. But how does it compare with simple finite difference methods in the three cases discussed? I think the authors could discuss the comparison in different situations; e.g., when one cares about only a few snapshots, and when needs to know the whole history of evolution. Having a fair discussion about the disadvantage of the method does not make the paper less significant, but instead show future directions where improvements can be made.

2, Lines 45-50: These listed numerical codes use either finite difference type methods or SDE/layer methods. None of the methods are limited by the choice of coordinates. Those authors chose a particular choice of coordinates probably because they did not intend to build a general library or because they tended to demonstrate a new method. I think it would nice for the authors to take this into consideration when discussing previous models.

3, Lines 53-55: The complicated geometry is a problem for some of the models, mainly because radiation belt people seem to have a preference for finite difference methods. There exist general powerful finite volume/elements methods that can handle complicated boundary geometry. So here "... would be challenging for numerical methods" should

really be "... would be challenging for finite difference methods."

4, In Table 1, the UBER library input, can the current code handle time-dependent D? I know the method can, but not sure if it has been implemented by the code.

5, Lines 158: No, the grids in the layer method need NOT to be uniform. It was simply chosen for simplicity for estimating error and demonstration purposes.

6, Lines 155-159: Yes, in SDE methods, one can design this kind of method to obtain the global solutions of  $f$  and its history. However, the key to implementing this in SDE is actually about finding an appropriate interpolation method. "The only operation ... would be interpolation" sounds like finding such an interpolation method is easy, while in fact it is NOT. For the described choice of irregular domain or nonuniform domain, the interpolation method still needs to be of high order to reduce systematic error from interpolation, and to preserve positivity of the solution. For example, if a simple 4<sup>th</sup> order polynomial interpolation method is used, one might introduce oscillation of  $f$ , and hence negative values of  $f$ , from interpolation, and hence violates one of the key advantages of the SDE method. That is why Tao et al., 2016 (doi:10.1002/2015JA022064) introduced those higher-order positivity-preserving methods to be used with layer method.

7: Lines 310-315: There actually exists flux-limited Lax-Wendroff methods that can avoid introducing unphysical negative solutions. And in the case of forming steep gradients, it is well-known that one should add flux-limiters to Lax-Wendroff type methods.

Overall, I think this is a nice paper, and it would be an important contribution to the community of radiation belt modeling. I simply hope that the authors can give a more balanced description of the UBER library, especially when comparing with other methods.