Geochronology Discuss., author comment AC1
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Reply on EC1

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Author comment on "Combined linear-regression and Monte Carlo approach to modeling exposure age depth profiles" by Yiran Wang and Michael E. Oskin, Geochronology Discuss., https://doi.org/10.5194/gchron-2021-34-AC1, 2022

1. Linear fitting with muons.
We agree with the reviewers that it was inappropriate to claim that our method does not require any prior knowledge: erosion depth or rate must be estimated. What we meant to convey is that linear regression inverts for exposure age and inheritance directly, without needing to define a pre-set boundary or initial prior distribution for these. The erosion rate (or eroded thickness) is a required input for our inverse modelling approach.
Our motivation for introducing this approach is, first, to expand the application of linear regression to model exposure-age depth profiles. Prior published linear regression methods ignore muons and do not adequately address erosion. Our approach incorporates muogenic production, making it possible to estimate ages for sites with steady rates or a defined amount of erosion. As demonstrated in the discussion section, this method can provide exposure age estimations with high precision for profiles with less than 2 times attenuation length erosion, suggesting it may be applied for most surface exposure dating scenarios.
Our second motivation is that as an inverse approach, the least squares linear regression approach directly solves for age and inheritance, while treating the erosion rate/eroded thickness as an input instead of an output of the model. This characteristic makes it a convenient tool for exposure age estimation. It can be used with Monte Carlo sampling to explore the full distribution of possible ages and inheritance from the variation of input parameters (including erosion). Linear regression is also useful as a starting point for forward (e.g. Bayesian) models. Inverse-modeled age and inheritance may thus help researchers to tune the boundary values for the forward models to get better simulation results. Therefore, instead of replacing forward methods, we will argue that our approach complements forward methods. In addition, as an added benefit of our inverse model approach, the effective age Te provides a straightforward way to explore the trade-off between erosion rate and exposure age.
By comparing results from our least squares linear inversion with a Bayesian inversion using Metropolis-Hastings sampling, we find the age estimation results have similar precision for both methods, which supports the robustness of our approach. We are considering including this comparison into our revision.
2. Monte Carlo error estimation
The editor’s description of the procedure of our inversion and Monte Carlo simulation approach is correct. We would like to modify the procedure as follows:
1. Generate all the input parameters (10Be concentration, sample depth, eroded thickness, production rate, attenuation length, etc.) from pre-defined distributions.
2. Fit the data with eq. 10 to get Ten and inheritance.

3. Calculate the exposure age use Ten from step 2 and parameters generated from step 1 using eq. 11.
4. Repeat step 1-3 for many times, sampling the underlying distributions of each parameter, to produce a distribution of results.
In our inversion, we treat the erosion as an input of the model, therefore the uncertainty related to the erosion can be treated the same way as other parameters. We treat the erosion as a known value instead of an inversion output for several reasons. First, it simplifies the inversion process, so that one can inverse linearly in 2D space, and the Te value resulted from the first step could be used to explore the trade-off between erosion rate and exposure age. Second, though theoretically it is possible to find a unique set of solutions for erosion rate and exposure age for a certain depth profile, in practice, the sample uncertainties always exceed the resolution that required to define that unique solution. This means including erosion as an unknown is unlikely to increase the precision of the estimation results. As mentioned earlier, we are considering including comparisons of our least squares linear inversion and Bayesian inversion with pseudo depth profiles to demonstrate that a simple Monte Carlo sampling with two-step estimation can provide sufficiently accurate results.

3. Negative Inheritance
Negative inheritance estimations can be prevented from inversion process, and we have incorporated that into our earlier published work (e.g. Wang et al., 2020). However, we realized in the writing of this manuscript that those negative results, though physically impossible, are necessary for mathematical reasons.
For example, suppose we have a sample group with a true age of 100 kyr and a true inheritance of 0 atoms/g. Because of the uncertainties within the sample measurements, ideally, we want to have the estimated distribution of the exposure age to be centered around 100 kyr, with approximately half of the estimation older than 100 kyr, the other half younger than 100 kyr. The estimations younger than 100 kyr correlate to some positive inheritance, while those older than 100 kyr correlate to negative inheritance. The overall distributions of the inheritance should be centered around 0, meaning that approximately half of the estimated inheritance should be negative. If we require non-negative inheritance during the inversion, it will end up with deviated estimation results: the inheritance would center around a positive value, while the central age would be younger than 100 kyr.
We do not agree with Dr. Hidy's comment that it may be an overstatement and the effect of negative inheritance may not be an issue for other methods. We tested the two methods (least square linear inversion described in our manuscript and a Metropolis-Hastings Monte Carlo Bayesian inversion) using pseudo depth profiles with zero inheritance. We find that requiring non-negative inheritance during inversion leads to underestimation of the exposure age for both approaches. We are considering including this into the revision.
4. Scaling methods
Thank you for the suggestion. We will use a more complicated muon production model instead.