The paper « Estimation of depth-resolved profiles of soil thermal diffusivity from temperature time series and uncertainty quantification » presents a modeling study of near-surface thermal diffusivity of soil using the Bayesian method and validates its approach with synthetic experiments and field data collected at a site in Alaska.

General comments
The study is of high interest and the paper is very well written. Several points could be improved to help the paper gaining a broader impact and to make it clearer to a diverse audience such as one could expect with Earth Surface Dynamics. While the introduction shows efforts in clarifying the various modeling approaches, the authors are sometimes too straightforward on some aspects and may disconcert a part of the possible readers. Field data and field site would deserve a more thorough description. What kind of ground it is? What is the period of measurement? Which sensors are used? What are the weather records during this period? etc. The connexion between modeling experiments and field data is quite unclear. The author thank Vladimir Romanovsky for providing data but do not explain why they chose this dataset. Details on the soil characteristics, lithology, climate characteristics, the data collection approach, the choice of the dataset, the data characteristics, etc. must be provided.

Dear reviewer,
Thank you very much for taking the time to review the manuscript and provide your valuable feedback. We modified the manuscript to provide more details on the measurements we used to create synthetic dataset. The measurements were acquired at a site on the Seward Peninsula, Alaska during the summer (Romanovsky et al., 2020) and the autumn (https://doi.org/10.5281/zenodo.5465253) period. These datasets are from locations characterized by the absence of permafrost, the presence of tall shrubs, and decreasing organic matter content and porosity with depth. Thermal diffusivity values of the three layer model were assigned based on soil sample analysis performed at the same site and location.

We also clarified the link between synthetic experiments and field case studies in the introduction. Indeed, we perform synthetic experiments in which we infer soil thermal diffusivity and assess its uncertainty under different environmental conditions (i.e., soil
temperature gradients and fluctuations), length of sliding time-window, level of measurement errors, and temperature sensor geometries (Sect. 3). These experiments stress our method such to ensure the development of an approach applicable in a wide range of environments (i.e., warm-summer mediterranean climate in Sect. 4 and tundra climate in Sect. 5). Tundra climates, in particular during low soil temperature gradient, represent here a challenging scenario. In addition, we highlight that we test our approach in an Arctic environment because of the importance to have methods that can help improve the parameterization of soil thermal parameters and organic matter content in ecosystem models simulating the feedback from Arctic ecosystem to Climate change.

The broad significance of the study is also overlooked. Same words are repeated throughout the manuscript to point out the broad significance (e.g. carbon and water fluxes) but more details or discussion points against examples would have more impact.

We improved several sentences in the manuscript to be more precise on how the presented approach can help improve the carbon and water fluxes. Specifically, shallow soil thermal properties modulate the impact of changes in weather forcing (incl., air temperature, snow thickness) on soil temperature and thus on biogeochemical processes (incl., microbial activity, soil respiration), as well as on deeper temperature regimes (e.g., permafrost table depth).

The interest of this study is the reproducibility of the calculation and the tools and codes for these calculations are not provided. Please, provide more details or data for reproducing the study on other ground and if relevant provide the code. Otherwise explain why it is not provided.

We have deposited the forward model written in Matlab that compute the 1D diffusion equation in an heterogeneous medium according to Equation 3 in the paper at https://doi.org/10.5281/zenodo.6350359. The script provided has been validated (i.e., tested with synthetic data generated by analytical calculation). The code on the inverse model based on MCMC and written in Matlab is a proprietary software (https://www.pc-progress.com/en/Default.aspx?dream ). However, similar implementation has been written in python and freely accessible here https://github.com/LoLab-VU/PyDREAM . All this information has been reported in the "Code availability statement" at the end of the paper.

One crucial point remains unclear to me: the field data are collected in a permafrost ground but the modeling approach is not appropriate for permafrost modeling: the latent heat is ignored (see comment below) and freezing processes are not discussed. Furthermore results show positive temperature. These limitations raise a major concern about the modeling approach that encourages to reconsider the paper after major revisions if the authors can not clarify their approach.

Based on the reviewers’ comments, we realized that we did not explain clearly enough the reasons why in this study we are using a heat-conduction-based model without representation of advection and latent heat process, its advantages and limitations, and why we used it in an Arctic environment. We clarified this in the revised manuscript.

The thermal model we are using (heat-conduction-based model without representation of advection and latent heat process) is only adequate and used to simulate heat transfer during periods of time when conduction dominates advection and when temperature remains well above 0°C (above 0.5°C in our study). This means that in our case study in the Arctic environment, we only apply this model to time periods when the soil is entirely unfrozen along the probe and when rain events are minimal. We agree that estimating thermal diffusivity during freeze-thaw processes would require us to account for the effect of unfrozen water (Romanovksy et al., 2000). Still, we do not intend to do that in our
The main reason that guided our choice of using the heat-conduction-based model is that in many situations, hydrological boundary conditions are difficult to assess. Indeed, using a more complex model (including advection and/or latent heat) would require to force or parametrize the model with information that is much more difficult to collect than temperature. For example, rain precipitation and its partitioning between surface flow and infiltration is a major source of uncertainty that we would need to account for if we simulate soil moisture variations. Similarly, representing freeze-thaw process would require to assess first the water content in unfrozen soil and thus would require to measure or estimate several parameters. While we are aware that a few studies have been estimating thermal conductivity by including the freeze-thaw process, they have generally assumed the total water content constant, used additional datasets from intensive sites, or made assumptions given the site hydrological conditions (e.g., full saturation) (Jafarov et al., 2014, Nicolsky et al., 2010). Still, in our study we aimed at developing a method that can be applied at locations where only temperature data are present, and where soil water saturation and porosity is variable. Thus, we decided to use a heat-conduction-based model without representation of advection and latent heat process to reduce the number of parameters to be estimated or constrained (and hence the likelihood of non-uniqueness of solutions) while limiting the applicability of our model to time periods when the soil is unfrozen and the heat advection is limited (i.e., dry period considered for our case study in the Arctic environment).

Finally, it can be noted that our approach involves repeating the estimation of thermal diffusivity with a moving time-window that:

1. is as short as possible in order to enable the estimation of thermal diffusivity during time-windows devoid of significant water fluxes (e.g., advection) not represented in our heat-conduction-based model, as well as the detection of advection processes that may occur intermittently e.g., caused by percolation events. In this study (Section 4), we showed that when such events take place the misfit between the model and the data is expected to increase, and thus the moving time-window approach can be used to evaluate when our model lack hydrological process representation.

2. does enable the detection of changes in thermal diffusivity over time, presumably linked to changes in water content

Overall, while we agree that our model does not represent all the complexity of hydrological processes, we believe this approach is pragmatic for many applications, and reliable when applied under the above-mentioned conditions.

Introduction : The introduction is very well written and provides an interesting overview of existing modeling approach.

L 89-90: What does « numerous » means? What does « landscape » means when it is written later that all data are collected very close to each other in the same type of ground? The data are obviously not collected at a « landscape scale » but at a sub-hectometric or similar scale in homogeneous terrain. This remark points out the need to introduce the field site in more details.

"Numerous“ locations refer to the locations at which the temperature probes were inserted in the soil to record temperature time-series (up to 27 locations over 2.3 km² at Teller site in Alaska). We have added a sentence on this in the paper. We have also substituted the word “landscape” with “field site“ through out the paper to avoid confusion.

Similarly, about questions (1) and (3), I do not understand how the study tackles the question of « different environmental conditions » or « different locations across the landscape » by focusing on a single site, even though several dataset are collected at this site.
We have clarified these points in the introduction. The field applications to different environmental conditions is aimed to testing the method under different soil temperature gradients and fluctuations which reflect different weather forcing. Indeed, we apply our method to two different sites: one in a warm mediterranean climate in California (Sec. 4) and one in a tundra climate in Alaska (Sec. 5).

The field site study in California is used to:
1- test the repeatability of our approach (the soil temperature is measured multiple times within a 25 cm radius area)
2- test the ability of our method to detect changes of thermal diffusivity over time

The field site study in Alaska is used to:
1- test the ability of our method to detect changes of thermal diffusivity over time and space (soil temperature time-series collected from 27 location within the 2.3 km$^2$ area of the Teller site)
2-evaluate the link between estimated soil thermal properties and soil physical properties from soil samples.

Theory and method
The approach contains many steps. I would suggest that the authors add a diagram outlining the different steps and how they interact to make it clearer (section 2 and 3).

We added a clear description of the various experiments and their goals (see response to previous comment) in the introduction. We prefer to not add a figure given that most of this information can be provided with words and that there is no iterative feedback mechanisms between these various experiments.

L 110-115: are the latent heat exchanges ignored in the modeling approach? It is a crucial process in permafrost ground and it has to be accounted for!

We did not apply our method to soil temperature $<0.5$ °C (see our response to earlier comment on field application in a discontinuous permafrost system). We have already addressed this question above.

Section 2.4.2: how is the thermal diffusivity assumed? Based on which knowledge?

Thermal diffusivity values are chosen based on the values measured from laboratory analysis on soil samples collected at the site. A sentence has been added in the revised manuscript to clarify this.

Results
I think that the results of the soil samples analysis outlined in section 2.4.4 should be clearly described such as the modeling results. This would also help to link modeling experiments and field data.

We added more details on the soil laboratory analysis in the manuscript. Laboratory analysis involved drying the soil samples at 65 °C, recording dry weight, grind soil to pass a 2mm sieve, record weight of the >2mm and <2mm portion, and measure carbon concentration by combustion.

Discussion
It would be easier to follow with subsections. It seems that many paragraphs repeat each other and that the ideas are not well ordered.

We have restructured the discussion section and added the following subsection to make them easier to follow:
- Impact of environmental conditions on thermal diffusivity estimates
- Impact of temperature sensor characteristics and deployment geometry on thermal diffusivity estimates
- Spatial variability of thermal diffusivity and link with soil physical properties
- Temporal variability of thermal diffusivity

Conclusion
It must be more specific with clear statements about the findings and broad significance of the results. In the current state it rather looks like paraphrasing of general statements provided in previous sections.

We modified and added clear statements on the results in the first paragraph and discuss their relevance in the second paragraph.

I am still not convinced about the use of wording such as « various locations » knowing that all data come from the same site.

We have addressed this concern above.

Detailed comments
L 96: why is this 10% or 5% threshold relevant? Maybe providing examples of possible applications with such thresholds would be relevant.

These thresholds are relevant because is the accuracy at which thermal diffusivity can be at best be measured from the thermal analyzers on the market such as the TEMPOS instrument with the SH-3 dual needle from the METER Group. We have added a sentence in the revised manuscript to clarify this.

L 137-138: on which basis these diffusivity values were determined?

The Courant-Friedrichs-Levyor condition is computed considering the maximum value of thermal diffusivity that could be used in the forward modelling. In order to set this upper limit for soil thermal diffusivity, we have reviewed the literature and found that the typical largest value is about 3 mm²s⁻¹ (e.g., Farouki (1981); Andújar Márquez et al. (2016)). We have now clarified this in the revised manuscript.

L 142: some data are mentioned but they should be introduced earlier in the paper.

In Section 2 ("Theory and method"), we do not provide all the details about the data used since they are not needed at this stage. Instead, detailed info on the data are provided in Section 3 on "Data and field sites description".

L 198: what reference/manufacturer are those sensors from?

The digital temperature sensors (TMP117AIDRVR, http://www.ti.com/lit/ds/symlink/tmp117.pdf ) are mounted on interconnected printed circuit board and inserted in a ~ 10 mm outer diameter (OD) plastic tube filled with epoxy (Dafflon et al., 2022). We added more details in the manuscript.

L 207-208: on which basis are these values determined?

Thermal diffusivity values mentioned in Section 2.4.2 and used in the synthetic experiments are chosen based on the values measured from laboratory analysis on soil samples collected at the site. A sentence has been added in the revised manuscript to clarify this.

Fig. 1: what is the « true » thermal diffusivity? The one calculated from field samples
(Sect. 2.4.4)?

The "true" thermal diffusivities here refer to the values derived from soil samples and used to generate the synthetic temperature fields used in the synthetic experiments.

L 333: what are these visual observations?

The visual observations refer here to what was observed from the soil pit dug at the site. We have clarified this in the revised manuscript.