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## Reply on RC2

Nate A. Mitchell and Brian J. Yanites

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Author comment on "Bedrock river erosion through dipping layered rocks: quantifying erodibility through kinematic wave speed" by Nate A. Mitchell and Brian J. Yanites, Earth Surf. Dynam. Discuss., <https://doi.org/10.5194/esurf-2021-3-AC2>, 2021

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**Dr. Gailleton,**

**Thank you for your thoughtful review and constructive feedback. First, we will address the influence of different  $m/n$  values. We have added twelve additional simulations in which  $m/n$  values are varied between 0.3, 0.5, and 0.7. The results from those simulations are shown in the supplement but summarized in the main text. Overall, different  $m/n$  values do not seem to substantially change the dynamics we study when contacts are horizontal. Although erodibilities cannot be directly compared when drainage area exponent  $m$  values differ, these new simulations have the same  $K^*$  value ( $K^* = 9.50$  for the three simulations  $n = 0.67$  simulations and  $K^* = 0.125$  for the three simulations with  $n = 1.5$ ). Because the simulations have the same  $K^*$  value, the erosion rates in weak and strong rock types are about the same.**

**When contact dips are non-zero, however, river behavior depends more strongly on  $m/n$ . For example, in Figure 7c (a simulation with  $n = 0.67$ ) we emphasized that peaks in erosion rate within the weak unit increase in magnitude with drainage area. The new simulations we present demonstrate that the rate of change in the magnitudes of erosion rate peaks depends on  $m/n$ . For example, when  $m/n = 0.3$  the erosion rate peaks have a smaller range (e.g., from about  $6E/U$  to about  $17E/U$  in Figure S12a). When  $m/n = 0.7$ , the erosion rate peaks have a large range (from about  $4E/U$  to about  $26E/U$  in Figure S12c). Although Figure S16 shows that contact migration rates are still well represented by Equations 12 and 13 (formerly Equations 14 and 15), the influence of  $m/n$  on the covariation of erosion rates with drainage area is an important consideration for non-zero contact dips. For example, because  $m/n$  values influence the covariation of erosion rates and drainage area they may also influence both spatial contrasts in erosion rate and drainage reorganization. We now discuss these results in sections 3.3.1, 3.3.3, and 4.2.**

**We decided to bin results by drainage area for visual clarity in our figures (i.e., instead of having a dense cloud of all measured contact migration rates in Figures 4 and 9). To address the influence of our binning approach, we have added a new figure to the supplement. This figure is a version of Figure 9 with 20 drainage area bins instead of 10. The contact migration rates and estimated kinematic wave speeds are generally the same, but there are slight differences**

(e.g., the  $R^2$  value in subplot (b) changes from 0.37 to 0.41). We now point out this consideration in the main text (Section 3.3.3).

To improve the readability of the article, we have cut material from the main text (especially from figure captions) and moved some material to the supplement (e.g., much of sections 2.2 and 2.3). Our intention was to thoroughly address the topics discussed in the manuscript, but we agree that focusing on brevity has improved the article. We also added a section to the supplement detailing how we extracted and processed channel profile data (Section S1). As you suggested, we also (1) added a new subsection called "Motivations" in the introduction (Section 1.1), (2) shortened the description of Figure 2, and (3) homogenized our use of the term "channel steepness" (rather than stream steepness).

We appreciate your feedback regarding the scaling between drainage area and discharge in the area of Hanksville, UT. Unfortunately, we are not aware of any such work in the area that focuses on the broad timescales (i.e., geomorphically significant flows) pertinent to the stream power model. We have expanded our discussion of using Tank Wash as a potential real-world example in section 4.4. We emphasize that our intention is only to demonstrate how the methods developed in this study could be applied to a real stream. A detailed study of Tank Wash that makes more specific assertions regarding its properties (e.g., erosion rates) and history would require more data than are currently available (e.g., contact locations and dips).

We have expanded the discussion of units with randomly varying erodibilities and layer thicknesses (Section 4.2, in the paragraph discussing how rock strength contrasts can drive landscape transience). Although the simulations in scenario 2 only used three rock types instead of two, this scenario was meant to provide insight into how channel slopes would vary with any number of different units. Specifically, scenario 2 shows that the channel slopes and erosion rates in each unit will adjust to allow for a consistent trend in kinematic wave speed across the profile. The exact channel slopes and erosion rates required depend on the distribution of erodibilities among the exposed units. In Section 4.2, we now emphasize that the exposure of a much stronger unit could (1) lower the kinematic wave speeds across the profile and (2) alter the erosion rates in other units. River incision through units with widely varying erodibilities and thicknesses might cause streams to be in a constant state of adjustment, preventing them from truly achieving a dynamic equilibrium.

**Our responses to your comments below are shown in bold text.**

line-by-line comments

line 17: I don't mind the term "stream steepness"; however I would recommend using "channel steepness" as a more common alternative.

**Response: We have replaced "stream steepness" with "channel steepness" throughout the article.**

line 30: I would replace "and" by "or" as one could suggest stream power has been used in many other situations (all the chi-related works expressed in the stream-power referential following Perron and Royden (2013) as one example among many).

**Response: We did not intend to suggest these examples portray all uses of the stream power model. To clarify that there are many other implementations of the stream power model, we now begin this list with the phrase "... the stream**

**power model has been used in many applications including: (1) ..."**

line 46: the work of Lavarini et al. (2019, <https://doi.org/10.1029/2018JF004610>) also is a nice example of the consequences differential lithology can have on detrital analysis, beyond the sole difference in erosion rates.

**Response: Thank you for the suggestion, we have added a sentence discussing that reference.**

lines 52-56: I apologise for the inelegant suggestion, but I think this preprint (<https://doi.org/10.1002/essoar.10505201.1>) would be a relevant reference for the use of channel steepness to explore lithological variations and their implications in landscapes evolution (it is in the final stages of the peer-review process and I hope will be in an accepted form for the authors' revisions).

**Response: Thank you for the suggestion, that article is certainly pertinent to this work. That article does not seem to be published yet, but we expect it will be soon. At that point, we will include the reference in this article.**

line 65: Briefly add a couple of words to detail the  $k_{sn}$  extraction method (i.e. from S-A,  $dz/dx$  regression or else).

**Response: We have added a new section to the supplement detailing how we extracted channel steepness (Section S1).**

Figure 1: The unit of  $k_{sn}$  is  $m^{2\theta_{ref}}$  (where  $\theta_{ref} = m/n$  within the stream power law referential), also the value of  $\theta_{ref}$  needs to be reported.

**Response: We displayed the units of steepness there as meters because we are using a reference concavity of 0.5. We now clarify that the  $k_{sn}$  values in Figure 1 use a reference concavity of 0.5.**

line 75: Alternatively, studies using steepness to unravel landscape evolution could also misinterpret variations in channel steepness due to lithologic variations as erosion contrasts (e.g. knickzones) due to base level falls. The different set ups in Figure 2 could lead to different type of misinterpretations.

**Response: We agree and have added a sentence emphasizing the potential for such misinterpretations.**

line 97: "Here" ? Do the authors mean "in this contribution"?

**Response: Yes – we now use the phrase "in this contribution."**

Figure 2: Nice figure!

**Response: Thank you!**

line 124: "nonzero" should be "non-zero"

**Response: We have made that change throughout the manuscript.**

line 130: "upwind", do you mean you are calculating  $dz/dx$  in the upstream direction or with an explicit Euler scheme? Calculating slope in the upstream direction could have some numerical consequences (see Campfort and Govers (2015), <https://doi.org/10.1002/2014JF003376>).

**Response: We use “upwind” here to refer to the downstream direction (as used in Royden and Perron (2013)). We are not calculating slopes in the upstream direction.**

line 136: see my main comment about  $\theta$ .

**Response: Thank you for your feedback. To address the role of different  $m/n$  values, we have added new simulations to the supplement (as discussed above).**

line 164: I don't find it confusing, it makes sense to me!

**Response: I'm glad to hear that!**

line 180: Does  $\chi_{sp}$  vary within the slope patch? I guess it does not matter here.

**Response: Yes, there are many slope patches each with their own  $\chi$  values. The dynamics of these rivers (i.e., spatial variations in erosion rate due to contact migration) would likely complicate the dynamics of slope patch migration rates, however. We don't want to focus on those details further in this article as such details would detract from the focus of the study.**

line 201:  $k_a$  refers to relative time (10 ka = 10,000 years ago), I would suggest to stick with kyr and yrs for the whole paper.

**Response: Yes, I normally use that convention. When I was preparing this article, however, I remember reading author instructions saying to always use “a” for years. I cannot find those instructions right now, so it is possible my memory is inaccurate. I now use “yrs” throughout the article.**

line 202: This is a rather short time step. Any particular reason?

**Response: When erodibility contrasts are high, erosion rates can become very high (e.g.,  $E > 10 U$ ). We used a small time step to avoid numerical instabilities.**

line 291: Just to make sure, steeper reaches = higher  $k_{sn}$  or higher  $S$ ?

**Response: Higher slopes will lead to higher steepness values, although drainage area and  $m/n$  must be considered of course.**

line 294: It is also important to state that the non-linearity of the relationship increases with  $|n - 1|$

**Response: Thank you, we have added that clarification.**

line 300: I feel like it could be stated more clearly that  $n = 1$  is not numerically stable/representable with this equation.

**Response: Thank you, we now emphasize that point.**

line 308: Why would  $C_{HW}$  and  $C_{HS}$  be equal?

**Response: It is my understanding that in his review for Perne et al. (2017), Kelin Whipple suggested they should focus on the potential for equal kinematic wave speeds in strong and weak units. Darling et al. (2020) also focused on equal  $C_{HW}$  and  $C_{HS}$  (with Whipple being a coauthor in that article). Proceeding with the assumption that  $C_{HW}$  and  $C_{HS}$  are equal in these numerical models can be**

motivated either through (1) observing the spatial variations in erosion rate that occur in these models (e.g., Figure 3) or (2) geometric reasoning (e.g., Darling et al., 2020).

line 330: replace "you" by "one" or "we".

**Response: Thank you, we have made that change.**

line 345: It also assumes constant erodibility and layer thickness for each rock type?

**Response: Yes, it does. Temporal changes in the erodibilities and/or thicknesses of uplifted units would cause gradual adjustments in fluvial relief. We have expanded the discussion of these issues in Section 4.2. Such adjustments would generally be very gradual, however (i.e., the timescale to uplift the unit with a new erodibility or thickness across the profile). As discussed above, if a new rock type with a substantially higher or lower erodibility was exposed this new unit could cause an increase or decrease in the kinematic wave speeds across the profile, respectively (i.e., the moderate trend in  $C_H$  maintained across the profile would shift due to the new erodibility). Such a change would alter the erosion rates in each layer.**

line 452: It is not clear why these specific values of  $n$  are used.

**Response: We now emphasize that a wide range of  $n$  values are possible, but our intention is to provide a small selection of examples with  $n$  values less than or greater than one.**

Figure 9: The figure is difficult to read, especially the legends. Again, I wonder how sensitive the data is to the way  $A$  is binned.

**Response: We have reformatted Figure 9 and hope it is easier to read now. We also added a version of Figure 9 to the supplement with 20 drainage area bins instead of 10 (Figure S17). This example demonstrates that our binning approach does have a slight impact on our results (e.g., the  $R^2$  value for subplot (b) changes from 0.37 to 0.41). The binning approach was mainly used to improve the visual clarity of our graphs (i.e., instead of a dense cloud including all measurements of contact migration rates).**

Figure 10: the scatter plots are quite dense and difficult to read. Maybe smaller points, or unfilled symbols or another type of visualisation would make it clearer?

**Response: Thank you for the feedback. Our intention was not for the reader to extract insight regarding specific points within the scatter plots. Instead, we only intended for the reader to see that all points follow a 1:1 relationship between measured contact migration rates and estimated kinematic wave speed. There is some scatter, however, and we used the dashed lines to provide context for the magnitudes of such deviations (e.g., the measured and estimated data are always within a factor of 2 of each other).**

line 780: Generally accurate for numerically "perfect" data, I suggest it is important to note this.

**Response: Thank you for the suggestion, we have added that clarification.**

Figures S1, S2, S3 and S4: These figures are very difficult to read, I would really recommend to rethink their style. I am not sure one can extract relevant information from

them.

**Response: Thank you for your feedback. We have reformatted these figures to better convey our intended message. During the beginning of the simulations, the streams need to adjust from the initial conditions. The maximum elevations can gradually increase or decrease during this adjustment. The adjustment time is dependent on both the initial conditions and the rock-uplift rate (i.e., the time required for a contact to be uplifted across the fluvial relief). The final maximum elevation is not always obvious before the simulation has been run, however, as spatial variations in erosion rate can complicate that consideration (especially when dips are non-zero). The purpose of Figures S1-S4 is to show that we gave the streams enough time to adjust from the initial conditions. Figures S1c, S2c, S3c, and S4c all have one line for each simulation used – there is a lot going on in each subplot, but the important observation is that all simulations have a relatively narrow range in elevations (e.g., always within  $\sim 10\%$  of the final maximum elevation, rather than the large changes in elevation that can occur during the initial adjustment). We have expanded the discussion of these issues in section 2.3.**