Earth System Dynamics Discussions

Earth Syst. Dynam. Discuss., referee comment RC2
https://doi.org/10.5194/esd-2022-43-RC2, 2022

Comment on esd-2022-43

Anonymous Referee #2

Referee comment on "Emergent constraints for the climate system as effective parameters of bulk differential equations" by Chris Huntingford et al., Earth Syst. Dynam. Discuss., https://doi.org/10.5194/esd-2022-43-RC2, 2022

The paper "Emergent constraints for the climate system as effective parameters of bulk differential equations" by Chris Huntingford et al. provides a formal description of emergent constraints as parameters of large-scale partial differential equations (PDEs). In contrast to small-scale PDEs explicitly coded into Earth system models (ESMs), these large-scale PDEs are not directly included in the models, but emerge across ESMs when aggregated across larger scales. Huntingford et al. provide two example PDEs derived from simple thermal models. By assuming different bulk parameters (e.g., heat capacities) for the different ESMs, they show that these PDEs can be used to derive emergent relationships between short-term and long-term responses of the system, which ultimately can be used as emergent constraints with appropriate measurements of the real Earth system.

General Comments

This paper reads well and provides an interesting approach that allows the derivation of emergent constraints from bulk PDEs. I agree with the authors that an emergent constraint discovery method based on physical reasoning and mathematical models is much more desirable than data mining, and will eventually lead to more credible and robust emergent constraints. However, I have some concerns about the relevance of this study regarding "real" emergent constraints.

Currently, a large part of the argumentation of the paper is based on two very simple PDEs. Especially in the context of a changing climate (which is a necessary condition here), I think the equations are too simplified. Since the PDEs are missing a "loss" term, a constant forcing will lead to an infinitely rising temperature, which is not realistic. For example, what happens if you add linear loss terms (linear feedback) $-\lambda*T$ to your PDEs (e.g., so that your eq. (2) is similar to eq. (1) of Cox et al. 2018)? Could you still derive the emergent relationships from these new equations? I can imagine that there are certain conditions (e.g., small times, small λ, large forcings, …) under which your original equations are good approximations, but it would be good to guide the reader in detail through this process. Additionally, it would be very helpful if you can provide more details on these emerging bulk equations themselves and why they should be present in an ensemble of ESMs. Do you have any recommendations how to find such PDEs? An example with a real emergent constraint would also be incredibly helpful. All this will ultimately help the reader to gain more trust in your framework.

Finally, two technical comments: first, it would be very helpful if you could use continuous line numbers (and not start with "1" on every page) and also add line numbers to figure captions. Second, please consider depositing your code in a publicly accessible repository (e.g., Zenodo) to make your analysis more transparent and reproducible for other researchers.

Specific Comments

- P.2, l.30: Maybe add a reference here? E.g., Knutti et al. (2017), https://doi.org/10.1002/2016GL072012
- P.3, l.4: It would be more precise to refer to "observational" data here (alternatively "observation-based").
- P.3, l.12: A better reference for this might be Hall & Qu (2006), https://doi.org/10.1029/2005GL025127. You might also want to cite Allen & Ingram (2002), https://doi.org/10.1038/nature01092 here.
- P.4, l.1-2: It might be helpful for the reader to add the key conclusion(s) of the discussion of Fasullo et al. (2015) you mention here.
- P.4, l.29: I guess technically it’s a function of the total noise, so ε **and** η, not only ε.
- P.5, l.18: Required for what?
- P.6, l.15: It’s not only the data points (I guess by "data points" you are referring to the (x, y) tuples you get from the models?), but also the measurements that constrains the forcing element b.
- P.6, l.15-16: I think this sentence is not clear enough: "With the forcing uncertainties common for both short– and long–term drivers". You need to explicitly assume that b_i/H_{0i}=const across models; you should mention that.
- P.6, eq. (8): You might want to refer to Fourier’s law here.
- P.8, l.17: Why don’t you simply divide T(0, t) by sqrt(t) to get a y that is not dependent on t?
- P.12, l.10: I think this classification only applies to linear second-order PDEs, not to every PDE.
- P.12, l.10-12: Can you elaborate what you exactly mean by these "one-to-one mappings" and why this should be the case? This is not clear to me.

Technical Corrections

- P.3, l.19-20: The second part of this sentence is hard to understand, please rephrase.

- P.3, l.20-21: This sentence is also not easy to understand, please rephrase.
- P.5, l.10: I wonder if your notation would be simpler if your variable t represented seconds, not years. Then you could absorb the seconds-per-year factor into the frequency ω and drop all the primes for the heat capacity altogether.
- P.5, l.26: There is a "." missing after “Eq”.
- P.8, l.22: There is a "." missing after the end of the sentence.
- P.11, l.5: It would be good to add a name for the symbol epsilon here, maybe “error term” or similar.
- P.11, l.16-17: Something is wrong with this sentence.
- P.14, l.17: This reference points to a preprint, please update with the published reference.
- Caption of Fig. 1: I think there is a word missing after "This response contains a seasonal (x axis) and long–term (y axis, with seasonality ignored)".
- Caption of Fig. 2: "seasonal" forcing instead of "season" forcing. Second to last line: the "measured" value of ΔT_S.
- Fig. 2: The argument in the cosine of the response term has a different sign than eq. (10). This does not matter due to the symmetry of the cosine, but should be identical to have a consistent notation.
- Fig. 2: The square root in the denominator of the second part of the response is missing. Same for the x and y axis label in (b).
- Figs. 1 and 2: The index "p" is missing for the heat capacity. In addition, sometimes the prime is missing.
- Fig. 1 and 2: Why are some parts of the formulas underlined?