Reply on RC1
Mikhail Verbitsky and Michael Mann

Author comment on "ESD Ideas: A Global Warming Scaling Law" by Mikhail Verbitsky and Michael Mann, Earth Syst. Dynam. Discuss., https://doi.org/10.5194/esd-2021-87-AC3, 2021

Dear Referee 1, Thank you for your review. The following is our response to your comments.

Comment: In this manuscript the authors suggest that the evolution of the global temperature is governed by two time scales of positive and negative feedbacks, external forcing strength and evolution, as well as time. Forming non-dimensional groups, time is normalised and an equation for the temperature evolution (2) is derived. Then different cases are discussed where first positive feedbacks dominate, leading to stable climate states, and when positive feedbacks dominate leading to climate instability.

Answer: With the exception of highlighted misprint (should be “negative”) you correctly captured most of the paper content. Unfortunately, in your brief overview you didn’t mention the third case – a climate having property of incomplete similarity. This is the critical part of our study for a number of reasons including a perspective of novelty that you raised below.

Action: No action is required

Comment: This manuscript is problematic in countless aspects, and I really don't know where to begin, and to end. Although I don't regularly use it, I am familiar with Buckinham's pi-theorem, and feel confident enough to know that if you provide it with a poor hypothesis, then the result will not be more insightful or fundamental. It does make sure the formulation is independent of the units used, but it is not magic.

Answer: We definitely agree that a physical hypothesis is a cornerstone of any study.

Action: In the revised manuscript we have sought to better clarify what the underlying physical hypotheses are in the current study, which is really more of a “think piece” (that’s all that is really possible within the tight 2-page length constraint of ESD Ideas).

Comment: A guiding principle in the climate sciences is conservation of energy, at least Arrhenius (1896) used it, but probably also earlier studies. Arrhenius realised that studying the energy balance at the top of the atmosphere was a useful starting point and identified forcing from CO2, negative temperature feedback and positive water vapour and
surface albedo feedbacks. Not using this as a starting point requires justification.

**Answer:** Obviously, conservation of energy is a paramount principle, but starting with the hypothesis (1) does not mean that this principle somehow is not observed. Frankly, we internally debated how to better introduce the hypothesis (1). One of the options we considered was to review an energy-balance model (it is currently equation (8)) and to use it as an “inventory” of the governing parameters. We decided against this approach because it may create an impression that our analysis is based purely on the energy-balance model like (8). This is not the case because equation (8) does not have a property of incomplete similarity. We wanted a more general approach and decided in favor of equation (1). We can see now that complete avoiding mentioning of the energy conservation may create confusion.

**Action:** We will articulate our approach better in the revised version of the paper.

**Comment:** Typically, it is found that a two-layer formulation of the climate system with a shallow atmosphere+ocean mixed-layer reservoir coupled to a deep ocean provides a good starting point (Hansen et al. 1985), and this type of model is used with slight modification in countless places, and it is well-justified (e.g. Gregory and Forster 2008). Does the here discovered theory in some way predict aspects of climate change that the simple two-layer model does not?

**Answer:** The purpose of our study wasn’t to introduce the entire hierarchy of models of increasing complexity (particularly within the two-page constraint of ESD Ideas). Instead, it is a thought piece that investigates different low-dimensional descriptions of the climate system. The zero-dimensional EBM, for example, is the simplest. From there one can of course build all the way up fully coupled three dimensional ocean-atmosphere models with interactive carbon cycle and cryosphere, etc.

Yes, it does. Because of its simplicity, our approach is insightful. We suggest that the “hot-house” climate may be preceded by a climate having a property of incomplete similarity. To our knowledge, this is a new proposition.

**Action:** No action is required

**Comment:** Usually, we would think of feedbacks as dependent on temperature only to first order (e.g. Sherwood et al. 2015). There are modifications to this, for example some feedbacks may change during a transient as the system equilibrates (e.g. Held et al. 2010, Geoffroy et al. 2013), or as the temperature changes (e.g. Bloch-Johnson et al. 2015). But these are higher order effects and the starting point is that the feedback scales with temperature. In their formulation the authors assume that there are negative feedbacks with one time scale and positive feedbacks with another time scale without any justification.

**Answer:** Yes, we consider two cases where positive and negative feedback scales are different. One case we relate to the recent climate history based on the simple zero-dimensional EBM experiments described by Mann et al (2014) and another case of the “hot-house” climate is hypothetical.

**Action:** No action is required

**Comment:** The paper by Steffen et al. (2018) appears to be used as a kind of confirmation of the theory. However, it is well known, and represented by the above mentioned theory, that if the feedback parameter becomes positive one enters an instability and a run-away climate, for instance snow-ball Earth (e.g. Budyko 1969). However, there is no evidence that the hothouse hypothesis of Steffen et al. 2018 is
correct as assessed by IPCC AR6.

**Answer:** We do not consider paper of Steffen et al. (2018) as a proof of our findings but simply as an example of possible regime where positive feedbacks may dominate over some range of variation (e.g., over the range where positive methane feedbacks play a dominant role - a process that ultimately saturates at some level of warming since the available carbon reservoir isn't infinite).

**Action:** No action is required

**Comment:** The other piece of evidence provided is the near linear temperature increase over a select period of time. But that is not proof.

**Answer:** We do not prove our scaling law but discover its parameters, a power degree. For a climate with dominant negative feedback, we use historical records to determine $m = -1$ and for a climate with dominant positive feedback we use equation (8) to find $m \geq 1$. Additional studies will be needed to prove (or challenge) our scaling law.

**Action:** No action is required

**Comment:** Overall, though, what I find most problematic with this manuscript is that there is no attempt made to connect with relevant studies. A proposed idea can be radically different from whatever is already there, an attempt to reinvent the wheel, if you want. But authors need to do their homework and explain why they did what they did and how that is different and potentially better than existing approaches. It is therefore not possible for me to recommend publication.

**Action:** Our presentation is very brief because of the 2-page ESD Ideas format, but we will revise the manuscript, within these length constraints, to clarify some of the points raised and responded to in this review.

Mikhail Verbitsky and Michael E. Mann