

Earth Syst. Dynam. Discuss., referee comment RC2  
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## **Comment on esd-2021-56**

Anonymous Referee #2

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Referee comment on "Inarticulate past: similarity properties of the ice–climate system and their implications for paleo-record attribution" by Mikhail Y. Verbitsky, Earth Syst. Dynam. Discuss., <https://doi.org/10.5194/esd-2021-56-RC2>, 2022

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### **Summary:**

Overall, the goal of this manuscript is to demonstrate that a reduced order model of ice sheets exhibits incomplete similarity. I will be honest that I found this study to be hard to follow. I apologize to the authors in advance. if misunderstood what they did or said. Based on the difficult I had following the approach, I might not be the right person to review this manuscript. Nonetheless, my comments are below.

### **Comments:**

The basic system of equation is presented early on. It would make the manuscript much more accessible to provide an expanded description of the model and the physical interpretation of the parameters. For example, the parameter "a" is described as a snow precipitation rate. But the snowfall rate depends on the climate. Glacial cycles are known to be drier than interglacial cycles. And does the snowfall rate also include the melt rate? Or is that specified separately? Clearly, the melt rate has to depend on climate doesn't it? And then there are a host of "sensitivity coefficients". What do these physically represent and how would I measure them?

My next question arises from the assertion of incomplete similarity and description of what this means. Now I am vaguely familiar with similarity and incomplete similarity. The authors first assertion is that the period of the system only depends on two non-dimensional numbers. Here it would be helpful to provide estimates of the physical magnitudes of each of the parameters based on whatever observations are available and to provide a physical interpretation for the "V-number" and why this controls the period.

But I think my biggest question is I cannot follow the connection between the period doubling and incomplete similarity. The typical definition of complete similarity is usually that the similarity function becomes independent of some non-dimensional group in the limit that the non-dimensional number tends to zero or infinity. By contrast, the definition of incomplete similarity is that the similarity function does not become independent of the non-dimensional group as the group tends to zero or infinity. Instead, you end up with a scaling law where the scaling function becomes proportional to the non-dimensional group to some power. As the authors note, it is not usually possible to determine the scaling power by dimensional analysis alone. In the exposition (line 94), neither of the parameters tend to zero or infinity. The one parameter is 1 and the other is 0.75, neither of which can be considered large or small compared to one. So then the question: what does this have to do with incomplete similarity? When I have done calculations to determine incomplete similarity the goal has usually been to determine the scaling exponent, but I am uncertain if the authors even tried to find the scaling exponent. I apologize to the authors if I misunderstood the analysis or their approach. Maybe I'm coming at it from the wrong direction.

I also did not understand the figures provided. The x-axis and colorer aren't labeled and the y-axis doesn't have units. What are we supposed to see here?

There is another subtle issue with the analysis which is that it is always difficult to determine if the behavior of a mathematical model is a feature of the simplifications of the mathematical model or is common to the more nuanced physics that is more representative of the physical system. Here it is unclear if the authors are claiming that their simplified model obeys incomplete similarity or if the general ice-climate system obeys incomplete similarity.

