Dear Samuel Kristoffersen,

Thank you for the positive comments on the submitted paper. We appreciate this feedback and will incorporate your comments and suggestions in the revised manuscript. Below we attach our replies to the raised concerns.

Comment:

Given that the two algorithms provide different vertical wind results, it is not clear to me which are considered the 'correct' results. My understanding is that the 3DVAR-DIV is being demonstrated to show de-biased horizontal and vertical winds, and therefore this new algorithm is quite useful to the community. It is, therefore, my opinion that the paper is suitable for publication, pending some clarifications regarding, in particular, the Tikhonov regularization.

Reply:

We present three mathematical approaches to minimize biases related to the estimation of vertical winds from meteor radar observations. The first two methods are applicable to all meteor radars and provide the mathematical justification of neglecting vertical winds in the standard meteor radar wind analysis. The synthetic data set is used to demonstrate the importance of such a bias correction assuming the challenging scenario of zero vertical winds, which is a non-trivial solution to the problem.

The second aspect worth emphasizing is that the vertical wind magnitudes depend on the temporal and spatial scales that a sensor has (observational filter), which poses additional challenges in comparing different instruments and analysis methods. The first two methods of a Tikhonov and generalized Tikhonov are demonstrated for a 300 km diameter observation volume, which is much larger than the 3DVAR-DIV volume of a single voxel of about 30 km. Thus, the expected order of magnitude for the vertical winds is supposed to differ as well by an order of magnitude.

More details are provided in the replies below.
Here are my major comments/questions regarding the content and the results.

**Comment:**

- Given that the two methods provide different vertical wind results (std~0.2 m/s for the Tikhonov regularization and ~1-1.6 m/s for 3DVAR-DIV), are the authors able to conclude which method is providing the correct (or most accurate) vertical wind results? This was not clear to me in the paper.

**Reply:**

The 3DVAR+DIV retrieval provides the most physically and mathematical consistent solution for the vertical winds and many sampling issues are resolved due to the much smaller voxel size. While there is no ground ‘truth’ that we can use to validate our retrievals to state which one is more ‘accurate’, we can conclude that the generalized Tikhonov retrieval for the monostatic systems provides reasonable statistical distributions compared to ICON-UA (large scales). For 3DVAR+DIV, there is no comparable independent observations at the small horizontal scales that it resolves (~30 km). Nevertheless, we consider both are valid estimates of vertical wind but suitable for different spatial scales. The smaller vertical wind estimate with Tikhonov regularization is expected because its corresponding larger horizontal scale (~300 km).

**Comment:**

- The Tikhonov regularization is tested on real data, and synthetic data with no vertical wind. Given that synthetic data is created using tides, planetary waves, and gravity waves, which should have vertical components, are these synthetic data physically realistic? Additionally, I think it would be worthwhile to do a test on synthetic data which has known vertical winds. Currently, the assumption is there are no vertical winds, and that is observed. But can the known vertical winds of synthetic data be retrieved? This would help to clarify if the small vertical winds are real, or simply the result of the assumption that the vertical winds are small.

**Reply:**

The scenario with negligible vertical wind is the most challenging one. The retrieval of small/tiny parameters is much more demanding than the retrieval of larger parameters/quantities such as the horizontal wind. The synthetic model that is mentioned in this study, was already incorporated in previous work to test the momentum flux retrieval (Stober et al., 2021, AG). This early paper did include non-negligible vertical velocities for the $<u'w'>$, $<v'w'>$ fluxes. So far, the model is physically realistic and pretty similar to what Fritts et al., 2010 demonstrated for SAAMER.

The algorithm is able to retrieve the correct vertical velocities for scales much larger than the observational filter. These vertical velocities are most likely in the order of a few cm/s and associated with atmospheric tides, planetary waves, and inertia gravity waves with horizontal wavelengths much larger than 500 km. We will refer to these waves as resolved scales. Small scale waves with 60 km horizontal wavelength and a vertical velocity magnitude of 1-2 m/s, will be recovered by the 3DVAR+DIV, but essentially result in $w=0$ m/s for the monostatic retrieval (generalized Tikhonov) with a 300 km diameter of the
observational filter. The unresolved scales are treated as atmospheric noise and are included in the total error budget (temporal and vertical shear).

Based on the retrieval algorithm, we are able to identify the largest issues to obtain the correct vertical velocities for the resolved scales. The algorithm will return the correct solution under the following conditions:

- all observations within a time-altitude bin are taken exactly at time $t_0$ and altitude $h$, which corresponds essentially to an infinitely small and short bin around our reference grid
- the radial velocity error must be much smaller than the nominal vertical velocity
- the vertical velocity field is homogenous within the observational filter

For the above-described conditions, the retrieval would recover the correct vertical velocity from synthetic data. However, this is basically never the case. Due to the binning in time and space, we usually tend to underestimate the vertical velocity in dependence on temporal and vertical bin size and the wave properties such as period, phase velocity, and horizontal and vertical wavelength.

The largest damage to the vertical velocity estimation occurs when large vertical bin sizes are used. Due to the strong shears caused by tidal waves larger, this procedure significantly increases the atmospheric noise caused by the vertical shear flow within a time-altitude bin and, thus, inhibits the ability to retrieve the correct values. Ideally, not more than 2 km vertical resolution with some small oversampling is advisable.

Furthermore, the retrieval tends to sustain the a priori state for large error observations and at altitudes with poor measurement statistics. The a priori is assumed to be $w=0\text{m/s}$.

**Comment:**

- Since the Tikhonov regularization effectively filters the vertical winds, are these results different than making the 0 vertical assumption, which is typically made with meteor radar winds?

**Reply:**

The first method presented in the paper involving a Tikhonov regularization is meant to provide a mathematical framework to underline that the assumption of negligible winds is not only just a quick idea, or simplification. The assumption actually has a solid mathematical and physical reason considering the spatial and temporal sampling. It is possible to show that a least-square solution for horizontal winds with the assumption of a zero vertical component, could be used as regularization constraint to fit in a second step for the vertical wind component. As zero vertical wind was assumed, it is expected that this procedure should return a small vertical wind magnitude as a solution. However, the histogram will be identical to the one presented in Figure 1. In summary, the assumption of a negligible vertical wind is better than to apply an ordinary least square fit for all three components, which would not only bias the solution for the vertical wind, it also has adverse effects on the horizontal components.

**Comment:**
Regarding the apparent motions of the specular scattering point, you mention that the radial velocity measurements are representative of a short time period (line 472). For times of less than a second, I would expect the air parcel motion to be no more than a few 10s of m/s. If the size of a voxel is on the order of kilometers, do these scales result in significant deviations in the observed radial winds?

Reply:

The apparent motion of the scattering point is not critical compared to the voxel sizes. As mentioned above the trail drifts only a few meters, which is not adding an issue for the 3DVAR+DIV retrieval concerning the localization of a meteor echo within a certain voxel.

The main issue of the apparent motion of the scattering center is related to the estimation of the vertical velocity. For monostatic systems, the Bragg vector magnitude (radial velocity) is affected by only a few cm/s, which is not a big problem for the horizontal winds but induces a bias in the absolute vertical velocities. These are usually in the same order of magnitude or less.

However, for forward scatter systems such as CONDOR, the apparent motion can become much more significant and reach several meteors and, thus, the Bragg vector magnitude can be biased by up to several m/s, which leads to significant limitations deriving horizontal winds. At the lowest altitudes of Figure 3, we obtain a factor of 2 too large horizontal velocities.

The main reason is the specular scattering along the meteor trail. This is like looking into a mirror. If you are directly in front of the mirror small motions of the mirror towards or away from an observer won’t lead to big changes in the image (represented here by the Bragg vector), while when looking at very slant angles at a mirror, small changes of the mirror position can lead to rather significant changes of the image.

We thank the reviewer for all the technical corrections and will include them in the revised paper and provide a point-by-point reply on the changes.

References:
