

Clim. Past Discuss., referee comment RC3  
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## Comment on cp-2021-136

Anonymous Referee #3

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Referee comment on "Orbital insolation variations, intrinsic climate variability, and Quaternary glaciations" by Keno Riechers et al., Clim. Past Discuss., <https://doi.org/10.5194/cp-2021-136-RC3>, 2021

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The paper is generally well written and will be a valuable resource for references on specific ideas related to simple models used for understanding the climate variability in the Earth System from the millennial time scale to the million year time scale during Pleistocene ice age cycles. Also the paper brings the reader up to speed on recent theory on autonomous, non-autonomous and random dynamical systems. I like how the authors use very simple systems to illustrate the pertinent dynamical systems and attempt to illustrate the pullback approach and pullback attractor with selectively chosen simple models. While I don't consider there to be any significant advancement in our understanding of millennial scale climate variability or the Mid-Pleistocene transition, there are some incremental improvements over the previous literature in that the proposed lower-order dynamics are illustrated very elegantly. I really only have some minor comments that should be relatively easy to address.

Comments and typos:

At lines 102 (section 1) the authors bring in the notion of a Hopf bifurcation with one type of simple system (eqn 5). Then in section 2.2 the description of the subcritical and supercritical Hopf bifurcations are described with another system (eqn 6). I would like to see more diagrams in Section 2.2 (some of us can visualise in our head what is happening when parameters are varied (e.g. through a Hopf bifurcation) but I think it is important to try to improve section 2.1 and 2.2 in a more unified way so as to make these sections more accessible to a newer audience that is reading this type of material for the first time. I

think a clear illustration with both language and an additional set of figures (possibly using the example systems from equation 5 or 6) would be helpful. For example, one might introduce the sections with language such as, "A Hopf bifurcation occurs when a periodic solution or limit cycle that surrounds an equilibrium point appears or disappears when a (control) parameter is varied. When the stable limit cycle surrounds an unstable equilibrium point, the bifurcation is supercritical. In the case that the limit cycle is unstable and surrounds a stable equilibrium point, the bifurcation is subcritical." And then also illustrated these concepts later on with the simple systems used.

In general the paragraphs are quite short (e.g Line 54). There is no need to start a new paragraph in a lot of places in the manuscript, please try to make the text flow a bit better.

L243, 249 monotonic, monotonically

I like the section on the pullback attractor and the figures used for illustration of the concepts in this section and therefore this section is a useful resource for general audiences.

In the section on 3.3 on applications D-O events.

Figure 8a is a bit confusing, maybe I missed it, but I don't see how the abscissa and ordinate are defined, it looks like simply  $x$  and  $y$ , yet they are both scaled to  $\alpha$ ? For example at line 371, the description of the  $\gamma$  and the fixed points that arise. I don't see any description on how the  $y = \gamma$  nullcline intersecting the cubic polynomial  $P_3(x,y)$  (manifold) is what determines the unstable or stable fixed points of the system. There are a lot of  $\alpha$  symbols illustrated on Figure 8a but there is no clear description in my opinion.

In Figure 8c the authors show the non-autonomous forcing for  $\alpha(t)$  and then on line 378 they introduce the non-autonomous  $\gamma$  and it is not clear what physical implications that  $\alpha$  provides although  $\gamma$  is related to CO2 eventually, and how the two non-autonomous forcing are related. The authors have not referenced or discussed how this work relates to or improves upon the work of Roberts and Saha (2016) which also illustrate non-autonomous dynamics on a FHN type model. In particular Roberts and Saha draw particular attention to how they modulate the slow manifold through time dependent changes and attempt to relate it to physical mechanisms (e.g. insolation forcing). They also introduce the time dependent sinusoidal forcing on the linear nullcline in the slow component of the slow-fast system. I'm not sure which processes are more important in attempting to explain the last glacial cycle millennial scale variability; either through an amplitude modulation of the slow manifold based on obliquity paced variations as Roberts and Saha have done or the time dependant variation of the linear nullcline using CO2 as the authors have done here.

The legend for  $t_0, t_1, t_2, t_3$  in Figure 8 looks incomplete.

I like figure 8b , I would almost like to see the 4 curves illustrated separately as the red dominates. I'm not sure if there is an easy way to do this.

I like the section on the MPT , but the additional value seems to come from the incremental understanding achieved from relating it to more recent concepts from NDS and RDS.

I don't particularly think the title is completely appropriate , but I don't have a good alternate suggestion. The authors mention orbital insolation , but there are also internal mechanisms, plus a lot of discussion on NDS and RDS , but I'm not sure you can formulate this into an adequate short title.

To summarise, I think the authors have done an admirable job in bridging different timescales of Pleistocene climate variability using more recent mathematical concepts and adequately simple dynamical systems to illustrate their points.