The paper is well written and describes the employed methods in sufficient detail. With the last section, the method is applied to a practical example and the differences are examined.

The topic fits the journal well.

The method, as extended in Sect. 2.3 is particularly useful when joining satellite measurements taken on different grids and with co-location errors, where only certain diagnostic matrices are provided. I suggest publication after properly positioning the CDF method as multi-variate inverse CM-weighted mean and addressing the major and specific comments below.

MAJOR COMMENTS

line 76

I think it might help here the understanding to introduce the relation of \( \hat{x} = A \cdot x_{\text{true}} + (I-A) \cdot x_a + G \cdot \epsilon \) as this shows more readily the nature of the formula: a weighted average of the
true state transformed by the different measurement characteristics of the involved instruments:
\[ x_f = (\sum S_i^{-1} A_i + S_a^{-1})^{-1} (\sum S_i^{-1} (A_i x_{true,i} + G_i \varepsilon_i) + S_a^{-1} x_a). \]
which also leads pretty naturally to the derivation of the aggregated averaging kernel matrix.

The formula above as well as (10) - (12) can also be simplified drastically by exploiting that
\[ S_i^{-1} A_i = F_i, \]
which mathematically is very reasonable and fits well to the general framework of optimal estimation and Kalman filtering.

Is the whole method, in its given form, not fully identical to a "simple/straightforward" linear optimal estimation/maximum likelihood estimate of all involved instruments *linearized* around the individual solutions? Which is indeed very reasonable, but not really a "new" method.
The new mathematical description makes this pretty obvious in contrast to the original, more convoluted formula.

The given mathematical notation can be argued for due to the information supplied by typical retrieval products, but both forms, the "standard" form using the (inverse) Fisher information matrix as weight in a weighted mean and the given form should be described and compared against each other.

The authors should discuss this and how it differs (or not) from the method described, e.g., by Rodgers in Sect. 4.1.1.

SPECIFIC COMMENTS
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line 33
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You stated that the method delivers the same result as a simultaneous retrieval, so in what respect or in relation to what can its quality be better?

line 48
Didn't you just state that the formula was introduced by Ceccerini (2021)? So it isn't introduced here, "only" discussed in greater detail?

In fact, Ceccerini (2021) seems to suggest that the formula was introduced by Schneider (2021)?

I think the historical development and relationship between the papers and methods should be discussed in slightly more detail than given here, taking into account in particular other peoples contributions.

Is the Python code with a reference implementation available? I.e. can the results of Section 3 be reproduced?

MINOR REMARKS

Who has proposed it?

Performances ... have -> performance has
I would say that while Rodgers provides a very useful discussion on the use of Kalman filters for the use case at hand, it is not a suitable reference without also giving (Kalman, 1960; see Rodgers). Are the references in lines 39 and 43 switched?

The readability of the formulas could be greatly improved when the "^\text{-1}" notation of the involved matrices would be above the index, not after it.