Comment on amt-2021-31
Thomas von Clarmann (Referee)

Referee comment on "Synergetic use of IASI profile and TROPOMI total column level 2 methane retrieval products" by Matthias Schneider et al., Atmos. Meas. Tech. Discuss., https://doi.org/10.5194/amt-2021-31-RC1, 2021

The authors of this paper are my colleagues and friends. Thus, I am not in a good position to provide an unbiased review. Due to this obvious conflict of interest, I would have preferred to provide neither a rating of the manuscript nor a recommendation, but the system forces me to do so. On request of the editor I have, however, checked the algebra in the Appendix. My rating of the scientific quality and my recommendation refer to the Appendix only.

The goal of this paper is to present a data fusion method that needs as input only data that are provided to the public, and thus can be applied by a wider community than the retrieval scientists involved in the level-2 processing of the parent data. The Appendix provides the theoretical basis for this.

My major concern is that in some places the derivations of the equations used seem to involve the inverse of a singular matrix. Such manipulations of equations are not truth-preserving. So I am afraid that it would need a major revision of Appendix A to remedy this. I have discussed this issue with the lead author, who meanwhile has found a theoretical basis which seems robust to me. Thus, I am confident that in the revision of the manuscript this defect will be remedied.

Some further issues:

Please note that the $S_x$ matrix in Eqs A4 and A6 has a probabilistic interpretation only if $S_x^{-1}$ is chosen as a regularization matrix. The text before A6 suggests that the authors consider also other constraint matrices. In this case, however, $S_x$ has no probabilistic interpretation.
lines 602/603: I do agree that Kalman filtering and retrieval use the same mathematical formalism, but I do not find it so clear in the comparison between Eqs A2 and A10. Formalism A2 is based on Rodgers' n-form, while A10 is more related to Rodgers m-form; the equivalence is correct, but it is not so obvious from these equations. Note that A10 involves covariance matrices, while A2 involves inverse covariance matrices. There is nothing wrong with the algebra here, but I would suggest to change the text.

B2 is not quite correct; we need an L matrix with x-values down the diagonal, and a \( ^L \) matrix with \( ^x \)-values down the diagonal. With these B2 should read
\[
A = ^L A^L L^{-1} = ^L A L^{-1}
\]
I think that the approximative character of this transformation needs to be mentioned, and the `approximately equal' sign should be used.

typo in line 661 "logarithmic" -> "logarithmic"

B3 can cause trouble: an \( S \) matrix representing a symmetric pdf in the log-space cannot be mapped on an \( S \)-matrix representing a symmetric pdf in the linear space. Thus, l661 should read "can be expressed in linear approximation in the linear scale" and the equal-sign in B3 should be replaced by an approximativly equal sign.

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