Thank you for sharing this work with the community. This paper represents good progress in the demonstration of the mathematical machinery which can be applied to the FORUM mission radiance dataset.

The main problem I see is that the term OE is used many times but the actual results do not include a climatology for surface emissivity so therefore this work is not Optimal Estimation nor is it the Maximum A Posteriori (MAP) method described in Rodger's paper. I believe it is more correct to refer to the method described as Minimum Information or Generalized Least Squares (GLS) using a diagonal matrix with a spectrally constant value. In other words this is the result you would get if you did not have any prior information about the land surface nor did you know anything about the spectral correlations of the land surface. For that reason it represents a kind of "worse case" scenario for retrieving information. As if you were flying FORUM on a different unknown planet somewhere outside the solar system. In reality the actual Earth's infrared surface emissivity is slowly varying both spectrally and temporally for any given latitude/longitude. The whole point of the FORUM mission is to provide the calibrated radiances globally for a long enough time (at least one year) that the radiance dataset can be used to derive a climatology of FarIR emissivity for a grid of latitudes/longitudes by month of the year. However the method described is far from optimal for doing that. For this reason I suggest that the abstract and conclusions be modified to indicate that the results are expected to greatly improve AFTER the FORUM mission is concluded and a climatology is derived (by some to-be-determined manner). Perhaps you can suggest some kind of timeline whereby currently you don't have a priori information so you can't use an OE (or MAP) method but by the end of the mission you hopefully can demonstrate an OE method that uses a priori information. This is just the first step and demonstrates that the machinery of the mathematics has been coded correctly and the somewhat disappointing results are expected to improve by the end of the mission.

Comments:

1) In Section 3.3 I suggest rewriting and expanding the first paragraph to state a bit more clearly the role that the xa and Sa play in the OE method. I disagree that the Xa is the a priori knowledge and the Sa is the uncertainty. I don't think that is a proper interpretation. In fact the Sa represents the variance of the a priori climatology so for surface emissivity this is the natural variability of the actual surfaces included in the
dataset. In particular, if you restrict the climatology to Snow/Ice scenes (by using some image classification for example) then the Sa matrix will "constrain" the solution so that it is consistent with the natural variability each spectra channel. This is critically important because as shown in Figure 7 the snow/ice emissivity is close to unity at 960 cm-1 and the variability is necessarily smallest there. In that case the Sa matrix will weight the spectral channels so that the fitted surface temperature is derived mainly from the 960 cm-1 region and that greatly reduces the error in the Ts. In that case the xa (guess) value at 960 cm-1 has a small variance and thus allows for the Ts to be derived in an true OE method with a physical constraint. This is how you narrow the "sloppy valley", kind of like a ford in the stream which provides the most obvious place to cross. At the same time the off-diagonal elements of Sa provide the spectral correlation for physical surface emissivities which is critical for estimating the surface emissivity in the FarIR using microwindows (mainly) that do not fully constrain the solution by themselves. So rather than just dismissing the entire purpose of the OE approach I think you could add some sentences that say why you want to use the OE method with a real climatology but you don't have one yet.

The issue you point out that a single global Sa matrix can not distinguish between quartz and snow is true but the solution is not to throw out OE the better approach, much better is to use scene classification and develop climatologies for each scene type or mixture of scene types. The approach for doing this "divide and conquer" approach is described in the following references and is already implemented in RTTOV 12+ for the MidIR. The same approach should work for the FarIR.


2) In the conclusions I suggest that you change the wording slightly to emphasize that the current results are what would be possible without an a priori climatology of surface emissivity BUT at the end of the FORUM mission you expect that the results will improve based on the development of climatologies that are consistent with the FORUM radiances.