

Reply on RC1

Sheng Li and Ke Du

Author comment on "A minimum curvature algorithm for tomographic reconstruction of atmospheric chemicals based on optical remote sensing" by Sheng Li and Ke Du, Atmos. Meas. Tech. Discuss., <https://doi.org/10.5194/amt-2021-122-AC1>, 2021

Response to comments of reviewer 1

We thank the reviewer for the helpful comments and suggestions regarding our manuscript. Listed below are our itemized responses, with the original comment/question displayed in italics. Please also find the revised manuscript with track changes.

General comments

The paper faces the problem to introduce the "smoothness" a priori information in the tomographic reconstruction of atmospheric chemicals based on optical remote sensing. In particular, a new minimum curvature (MC) algorithm is proposed and applied to multiple test maps. The performance of the new algorithm is compared with that of other existing algorithms. The MC algorithm shows almost the same performance as the low third derivative (LTD) algorithm but with significantly less computation time.

I think that the subject is correctly presented in the introduction and sufficiently put in the context of the existing literature on the argument; instead, I find that the description of the method is not given in all needed details. I suggest to improve the description of the method and below I give some suggestions.

I think that the paper deserves the publication on AMT after that the following issues are considered.

Thank you for this comment. We have improved the description according to the suggestions below.

Specific comments:

(1) *In the Tikhonov approach, an important issue is the choice of the value that is given to the regularization parameter, because this value determines how much a priori information goes into the results. In the paper, it is specified only "the regularization parameter is set to be inversely proportional to the grid length". I suggest describing the criterion that it has been followed for the choice of this parameter.*

This is a very good suggestion. We have rewritten the Sec. 2.2 to give a detailed description of the determination of the weights and regularization parameter. To summarize, equations are first assigned weights which are inversely proportional to path/grid length, in order to make sure that equations with different lengths have equal weights. In practice, the weights for the laser paths are set to be the same value (one), and weights for the derivative equations (regularization parameter) are set to be another value, which is to be determined. Then the regularization parameter is determined based on the commonly used discrepancy principle. For computational efficiency purpose, the

regularization parameter can be selected from several widely varying values due to the fact that the reconstructions vary only slowly with the regularization parameters. Finally, the one produces the smallest discrepancy is used.

(2) *It would be interesting to know if the algorithm is able also to produce a diagnostics of the results. Generally, a procedure that solves an inverse problem provides also an estimation of the errors (more in general of the covariance matrix) of the products. Furthermore, it would be useful to have quantities (such as the averaging kernel matrices obtained in the case of retrieval of atmospheric vertical profiles) that provide the sensitivity of the result to the true state, which are useful also to estimate the spatial resolution of the result.*

This is a good idea. The inverse problem in this manuscript is an over-determined system of linear equations, which can be analyzed using the conventional measures. However, the original inverse problem based on coarse grids is transformed to be a regularized problem with fine grids. In this case, only very limited number of grids are passed through by the laser paths. Others are restricted by the smoothness information around local neighbors. As a result, the residuals for each laser path can be easily minimized to be very small ($1e-4$), even though the results might be unrealistic. Therefore, traditional measures like goodness-of-fit or covariance matrix are not that helpful as usual situation. Meanwhile, the averaging kernel matrix (parameter resolution matrix) is a perfect identity matrix (we have checked this from the reconstruction results).

We think that this kind of inverse problems is different in some ways with the retrieval of vertical distribution of an atmospheric parameter from remote sensing measurements. In the latter application, the priori information of the state vector (a priori profile) is usually used. Then the averaging kernel matrix is an important measure to characterize the solution of the retrieval, including the retrieval resolution effects and a priori effects. But in the application of two-dimensional gas mapping, there is no a priori profile of the concentration distribution used. And the reconstruction is based on the high-resolution discrete grid pixels instead of weighting a set of profiles. Therefore, measures like nearness and exposure error are generally used to estimate the quality of the reconstruction in the field of computerized tomography or imaging,

(3) *Line 43: I suggest to put a reference for the Radon transformation.*
The reference has been added.

(4) *Line 141-149: In the description of the LTD algorithm it is not clear which are the equations of the system that has to be solved. I understand that for each cell we have two equations obtained setting to zero the third derivatives (in both direction x and y , I suppose, but it is not specified). Then, which are the other equations? Those obtained to look for the minimum of Eq. (3)? Please explain in detail which are the equations of the system that has to be solved.*

The reviewer is correct. At each grid pixel, two additional linear equations are appended. These generated equations for all the grids are then combined with the original linear equations (Eq. 2) of the laser paths, resulting in a new large system of linear equations, which is over-determined. The new system of equation is solved to find the concentrations. We have rewritten the contents describing LTD algorithm in Sec. 2.2.

(5) *Line 159-160: From Eq. (7) I understand that the seminorm is a number relative to the whole field, therefore, I do not understand the meaning that "the seminorm can be calculated at each pixel". Then, which is the summation mentioned in the text? I think that a more clear explanation is needed.*

Thank you for pointing this out. The reviewer is correct that the seminorm, which is the total squared curvature, is defined on the whole field. The total square curvature is the summation of the squared curvature at each grid pixel. To minimize the seminorm, we need its partial derivative with respect to the concentration at each grid pixel to be equal to zero. Thus, we get a difference equation at each grid, which is appended into the

original linear equation to form a new system of linear equations. A clearer description including the derivation process has been added to the Sec. 2.3.

Technical corrections:

(1) *The authors introduce many acronyms, but not all of them are then used. I suggest introducing only the acronyms that are used several times in the paper.*

Thanks for this good suggestion. We have checked the acronyms and removed those used only once.

(2) *Line 26: equality ---> quality*
Corrected.

(3) *Line 85: necessary ---> need*
Corrected.

(4) *Line 136: what is the superscript 21 after "problem"?*
The number has been replaced with a reference.

(5) *Line: 174: well-posted ---> well-posed.*
Corrected.

(6) *Line 212: It ---> it*
Corrected.

(7) *Line 242: increase ---> increases*
Corrected.

(8) *Line 286: equality ---> quality*
Corrected.