Reply on RC2
Maryam Ilyas et al.


Assessment: In the end, it seems to me that the authors come out with an incremental improvement on their own previous construction and it doesn't seem to be the "game changer" that the authors were possibly hoping for when they set out along this path. For example, the uncertainty bounds in Figure 5 are only very slightly different from those in their own previous construction, and the extended example on pages 9-11 seems to have been specially constructed as a worst-case scenario (as is explained in the paper, May 1861 was the month with the least spatial coverage of the entire dataset, therefore, presumably, the one for which the advantages of this kind of approach should be most clearly seen). Nevertheless, the work seems to be been competently carried out and it is always useful to have another dataset for comparison - I fully support publication.

Comment: My main concern about the paper is that the manuscript itself seems to have been put together in some haste. Prior to reading this, I was familiar with the LatticeKrig approach but not with the intimate details. Some of those details are important, e.g. the definitions of the lambda and aw parameters (page 4). I believe, throughout the paper, there is a need to give more explicit detail about the method. Given the 62 months of computer time, it seems unlikely that anyone else would want to exactly reconstruct this dataset, but nevertheless, I still feel that this should be a requirement of publication, that the method should be described in sufficient detail that anyone who wants to reconstruct the result has all the information required to do so.

In my initial correspondence with the editor, I queried why the authors had not produced a "supplemental materials" document - as I understand, such a document would be supported by the journal and I might suggest that the authors take advantage of this when revising the paper. Specifically, the kind of detail that is probably not relevant to the casual reader of the paper, but would be needed by anyone actually intending to try to reproduce the results, could very well go into an online appendix.

Response: Apologies for not properly defining the model parameter definitions. These details will be added to the paper.

Comment: One further "general" comment - there is a passing reference to HadCRUT5
(Morice et al. 2020), which is the latest version of the U.K. Met. Office model. I wonder if maybe the authors should say a little more about this. I presume the production of HadCRUT5 overlapped the present effort but would the authors like to comment on how HadCRUT5 improves on HadCRUT4 and specifically how it compares with the present work?

**Response:** Sure, this discussion will be added.

**Specific comments:**

**Comments:** p. 1 line 16 - "phenomena" (plural of phenomenon)

- 2 l. 15 Here the author Rohde is misspelled Rhode. This error also occurs in p. 2 l. 20, p. 3 l. 1 and p. 18 l. 24
- 2 l. 34 Reference to JMA (?) where the question mark is a standard latex warning for a missing reference. Were the authors referring to the paper Ishii et al, mentioned on p. 2 l. 14?

**Response:** Apologies. These will be corrected.

**Comments:** p. 4, l. 19. Here the authors mention two parameters from Nychka et al 2015, called lambda and aw, but they never define these two parameters. The implication is that one can look up Nychka 2015 to find these definitions but I tried doing that and I think we need more assistance.

Nychka (2015) defines a parameter lambda=\(\sigma^2/\rho\) but they don't call it a smoothing parameter - that was my first confusion. I do note that the LatticeKrig R manual also defines lambda and does call it a smoothing parameter - the most recent version of this that I downloaded for preparing this review was version 8.4 dated November 2019 (the authors of this paper refer to version 6.4 as the one they used for the bulk of their computations). If the present authors want to call it "the smoothing parameter" without further explanation, they need to be precise about where this is defined, and the answer appears to be the LatticeKrig manual (which I'll subsequently refer to as LK), not Nychka (2015) (henceforth N15).

**Response:** We will precisely define both the parameters and cite the LatticeKrig manual.

**Comments:** Both N15 and LK say that lambda and rho are computed my maximum likelihood and I understand that one of the objectives of the present paper is to extend that by using the ABC approach to approximate lambda, but what happened to rho? This isn't explained here, but later (p. 6 l. 4) they say, "both d and rho are still estimated using the maximum likelihood approach".

In fact d and rho are respectively an overall mean and a variance (scaling) parameter of a multivariate normal distribution and it is well known and trivial to implement that these parameters can be integrated out analytically, so as to focus attention on the spatial correlation parameters - in my first reading of the paper I assumed this was what they had done. But the comment on p. 6 l. 4 makes me wonder about this point. In summary, we need clarification of what the authors actually did. (If it really was a Bayesian approach, we also need to discuss the prior distribution, since rho in particular may require a proper prior.)

**Response:** Apologies for this confusion. The proposed model is not fully Bayesian. Only ABC posteriors of lambda and autoregressive weights were being determined. Other parameters (e.g. d and rho) were estimated by maximum likelihood.
Comments: Now let me turn to the other parameter, aw, referred to as "autoregressive weights". This is based on the fact that at each level of the multiresolution process that defines the spatial model, the coefficients of the radial basis functions have the structure of a lattice process that is assumed to be of conditional autoregressive (CAR) form. However, here is no fixed structure for this and no single parameter called the autoregressive weight (or weights - it's not clear whether the authors actually meant to use a plural form here). p.584 of N15 refers to a weight matrix B where the off-diagonal entries are -1 and the diagonal entries are of form $B_{j,j}=4+kappa^2$. In this case the lower bound 4 arises because the sum of entries in B is required to be positive. So is kappa the autoregressive weight? LK actually use a different notation, where they define a variable a.wght (this is the nearest I can find to any variable actually called "aw") and they comment (p. 23), "in the simplest case a.wght is the central value, and should be greater than 4". So is a.wght the same as $4+kappa^2$ in the N15 notation? If so, which parameterization do the present authors actually use? Later (p. 8 l. 21) the authors give aw a prior density that is uniform on the interval (1,4), but now I'm really confused about how that particular range was determined ...

Another potential wrinkle is that N15 p. 585 explicitly mentions the possibility that the autoregressive structure may be different at different levels of the multiresolution process, but I'm reading between the lines that they didn't consider that extension in this paper.

I don't actually think any of these questions are complicated. I understand very well that there are certain model choices that you just have to make. The authors simply need to be explicit about what these choices were and how exactly the various parameters are defined.

Response: The reviewer is well justified in asking for clarification about the exact form for the spatial autoregression (SAR) parameter that controls the spatial dependence among lattice points. Although there is the freedom to choose a different parameter for each level of multiresolution level in this work it is fixed at a single value. This choice is consistent with more traditional covariance models such as the Matern family in spatial statistics. As the reviewer correctly pinpoints, the a.wght parameter built into the LatticeKrig code for rectangular lattice points is the autoregressive parameter and is the same as $4+kappa^2$ in 2 dimensions, and in this form, the 4 nearest neighbors get a weight of -1 in the SAR formulation. Moreover, kappa can be loosely interpreted as a correlation range parameter for the spatial field. The particular application to a SAR model on the sphere and the icosohedral based lattice, however, led us to a slightly different parameterization. This is due to the fact that the lattice points can have different numbers of nearest neighbors. Twelve points will have 5 neighbors and the remaining points will have 6. Moreover, these points will not be exactly equally spaced. Given these irregularities, we found it easier to give a weight of $1 + \exp(2*omega)$ to the center lattice point and constrain the sum of the nearest neighbor weights to sum to the negative one. Thus, in this spherical version we take a.wght$= 1 + (\exp(omega))^2 = 1 + \exp(2*omega)$ and parametrize this value through omega. exp(omega) can be interpreted as an approximate correlation range for the SAR and omega itself is useful in representing the log of this scale parameter. In particular, priors put on a.wght are done through the assumption that omega follows a uniform distribution over the interval $[-4.5, .55]$. This choice covers a useful span of spatial correlations when omega is translated back into the a.wght parameter and subsequently into the dependence of the field at the lattice points.

Comments: p. 5. The flow diagram illustrating the ABC approach is clear and should be easy for the reader to follow, but again, some specific details are missing. How do they determine the number H of variogram sampling points and the specific vector of distances represented by h? I'm assuming that when you write gamma(h) you don't literally mean h as the distance (h is an index going from 1 to H) but gamma(d_h) for some distance d_h,
but then the same question, what values did you actually use and how were they chosen?

**Response:** All the standard rules are observed while computing the semivariogram. Semivariogram is computed up to half of the maximum distance between the points over the whole spatial domain. The distances are Great circle distances in kilometers. The number of bins (H) is 12 that is somehow standard. “h” is the center of each of the 12 lag classes that are defined over half of the maximum distance. These details will be added to the paper.

**Comments:** p. 6 equations (5) and (6). I don't think we need you to define every symbol here but please give an exact source for these equations, which I assume are somewhere in N15?

**Response:** These equations are slightly different.

**Comments:** p. 6 lines 16,17 - reference to a new dataset HadCRUT5 (which I wasn't aware of myself until reading this paper). I presume HadCRUT5 came out while this paper was being developed. I think this sentence should be moved to the discussion section and the authors should discuss how the two approaches compare and contrast with one another - are there any features in which HadCRUT5 improves on the present approach?

**Response:** We will add these details in the discussion section.

**Comments:** p. 7, l. 22 - "since the last few decades" - slightly awkward English construction here, maybe "during the last few decades" would be better. I am aware that the word "since" would be used in several other languages, for example "depuis" in French.

**Response:** These sentences will be rephrased.

**Comment:** p. 7, l. 28. Unclear why you set any negative value to 0.0. While I'm well aware that we all talk about global warming and not global cooling, I don't think the possibility of cooling is excluded by basic atmospheric physics - if a stochastic model occasionally produces a negative value, why not include it in the analysis? From a political point of view, the authors should take care to avoid any implication that their approach was predetermined to result in a warming outcome.

**Response:** These details are for Morice et al. (2012). We will cite Morice et al. (2012) and remove these sampling details from this paper.

**Comment:** p. 8 l. 28. This review is already getting rather lengthy and by this point, I was definitely suffering from reviewer fatigue, but if I'm not mistaken, this is the first time in the paper there is any parameter called alpha. Please, either define it or give an explicit prior reference where it is defined.

**Response:** Sure, we will cite references and additional details for this parameter.

**Comment:** pp. 9-11. The authors are quite explicit that May 1861 was chosen for this illustration because it was the month with the poorest spatial coverage, and therefore presumably the one that best illustrates the advantages of using a more refined spatial approach, but I think it would be helpful to have at least some comparisons with other months. Are these kinds of plots typical of what we would expect if we just chose a month at random?

**Response:** More plots in the appendix can be added for comparison purposes.
Comment: p. 10 lines 3-4: so part of the reason for the difference is that the LKrig function was improved between the two versions of LatticeKrig that were used for the 2017 paper and this one? Could you expand on that a bit - was that a major factor?

Response: More explanation will be added.

Comment: p. 12 l. 24 and Figure 5b: what exactly is the "median time series"? I'm inferring that each section of the time series was centered about its median value but what time scale was used for calculating the medians?

Response: That's true. Also, for calculating the medians annual time series were used.

Comment: p. 14, this is an additional feature that is only introduced later in the paper and somewhat complicated to evaluate. It seems that the authors do not intend to publish their full 100,000-member ensemble but only a subset selected by a conditional latin hypercube sampling (CLHS) approach? I'm sure there are good reasons for doing that but at least from the appearance of Fig. 6, there appear to be some nontrivial differences between the two approaches, or am I misinterpreting this figure? Once again, the fact that they have shown this figure only for May 1861 may in some sense be a worst case scenario, but it would be helpful to clarify that point.

Response: We would be happy to publish the full ensemble. We are looking at the options. Also, in the appendix, we can add figures for other months.