Comment: An interesting paper with a few issues to be resolved. In their analysis of global temperature data using the multiresolution lattice kriging method, the authors extend the work of Ilyas et al 2017 by exploring the hyperparameter estimation uncertainty using a Monte Carlo sampling method. It is interesting to see the impacts of this hyperparameter uncertainty assessment. These are a potentially important source of uncertainty in assessments of observed global temperature change that have not previously been investigated in other studies. There are some issues with the paper structure, including a lack of concluding remarks. Additional discussion of the effects controlled by the sampled parameters and illustration of the impacts of their sampling throughout the temperature record is needed.

Response: We are grateful for the encouraging comments. The section-6 discussion will be replaced by conclusion and discussion where concluding remarks will be added. The impact of infusing uncertainties in the parameters will be discussed more in sections 4 and 5.

Comment-1: While it is great that the paper includes estimates of hyperparameter uncertainty, I am left uncertain on the extent to which hyperparameter uncertainty translates into an appreciable uncertainty in the temperature fields and how this varies through the temperature record. The paper only provides examples of hyperparameter estimates and resulting fields for a single month. Is this representative of other months? Interpretation of differences between the ABC based and profile likelihood based analyses would be aided by showing how the profile likelihood hyperparameter estimates compare to those from ABC, for example in Figure Similarly, temperature/uncertainty fields are only shown for a single month in Figures 3/4.

These points seem important to understand the benefits of sampling the hyperparameter uncertainty, also given that the method appears to be computationally expensive.

Response: Uncertainties in the spatial field pretty much depend upon the spatial coverage. This single field was chosen to highlight the maximum impact of parametric uncertainties as it has the least spatial coverage. This point would be discussed in the paper. Additionally, comparisons of ABC and profile likelihood estimation procedures will be added.
Comment-2: The review of prior literature is frequently a few years out of date and needs updating. Some cited studies are inaccurately or incorrectly described. See detailed comments for details.

Response: Literature of more recent studies will be added. References will be corrected (apologies).

Comment-3: There appear to be a few simplifications in the statistical model/uncertainty model used that are not discussed:

Ø Hyperparameter estimates are global, estimated independently for each field, with no regional estimates, essentially modelling temperature anomaly variability as being identical at all locations over land and sea.

Ø As a space only model (not space-time) there appears to be no accounting for persistence of temperatures used to aid reconstruction or accounted for in uncertainty estimates.

Ø My understanding of MRLK is that it models observational error as identically distributed for each observed location. The analysis described makes no mention of the additional uncertainty terms for HadCRUT4 (in addition to the ensemble) that describe differences in observational error distributions for each grid cell and correlations in errors between grid cells, arising from the movement of marine measurement platforms. It appears that this information, that is not encoded into the HadCRUT4 ensemble members, has not been used and they are not described in Section 3.2. These uncertainties were found to be important in Morice et al 2012. Some comment on not including these, or how they are approximated by the additive uncorrelated error term in MRLK, would be appropriate.

Response: We apologize for our failure to sufficiently describe the statistical/ uncertainty model. We agree on the first two. For the third one, additional uncertainty estimates that are not blended in the ensemble members are not considered in constructing this data product. All these details would be mentioned in the paper.

Comment-4: The reader needs to refer to Nychka et al 2015 to understand the meaning of the lambda hyperparameter. The “aw” hyperparameter (please rename to use a single letter unless it is a product of two variables) does not appear to be described in Nychka et al 2015. I do not understand how to interpret the function of this “autoregressive weights” hyperparameter and how it might affect the resulting temperature fields.

Response: Apologies for this confusion. We will replace “aw” with a single letter. Autoregressive weight is a sort of range parameter that is used in the precision matrix of the Gaussian Markov random field. The definitions of model parameters will be added.

Comment-5: Discussion of Figure 5 suggests that uncertainty estimates for global average temperature anomalies are wider for ABC that those of Ilyas et al 2015, but this is not particularly evident in Figure 5. Uncertainty ranges appear to be roughly comparable, of similar magnitude to the quantisation in the plot of roughly 0.01 °C, but slightly skewed one way or another. It’s not clear that the ABC sampling of the hyperparameter uncertainty would necessarily lead to wider uncertainty estimates in the global mean than the profile likelihood estimates. For example, the lambda parameter is defined in Nychka et al 2015 as lambda = noise variance / process variance. Sampling into high values of lambda would give a process with low variance and large measurement noise, which would lead to smaller uncertainties arising from sampling limitations. The lower variance of the ABC analysis field in Figure 3 suggests that this might be the case. It would be an alternative explanation to the changes in LatticeKrig 6.4 that are alluded to in the first
Response: Thanks for the alternative explanation. Explanation in this context will be added in the paper.

Comment-6: The paper ends rather abruptly with a discussion of a sampling method (which arguably should be moved earlier in the paper). It would benefit from a conclusions section. Are there any deficiencies in the approach that we should be aware of? What’s missing or could further developed? It could comment on developments while this paper was being worked on that are not included, e.g. for HadSST4 and talk more broadly about where this study fits alongside other research in the subject area. It would be an appropriate place to place a link to the data.

Response: As described earlier, the conclusion and discussion section will be improved. Latin hypercube details can be moved earlier as well.

Detailed points.

Comment: Page 1, Abstract, line 1: Needs the word global in there to indicate that we’re talking about global temperature records?

Response: This will be changed.

Comment: Page 1, Abstract, Line 7: It’s not clear in the abstract what the “variation” in parameters is referring to. Hyperparameter estimates vary from month to month but not spatially. Or is this referring to the uncertainty in hyperparameter values, which otherwise isn’t stated in the abstract and is the key addition in the paper?

Response: These are uncertainties in the parameters. This will be clarified in the paper by modifying line-7.

Comment: Page 1, line 15 – Good et al 2016 is a satellite-based skin temperature record, not air temperature?

Response: Apologies. We will correct it.

Comment: Page 1, line 19 – It’s not exactly so simple as obtaining data from the WMO/GCOS. Modern messages are transmitted via these means but much work is required to compile observations from individual nation states and from research institutions to compile the historical records.

Response: We will improve this line.

Comment: Page 2, line 11 – The most recent version of the NOAA record is now called NOAAGlobalTemp with the following reference: ..... 

Response: Thanks. This line will be modified and the new references mentioned will be added.

Comment: Page 2, line 14 – Ishii et al., 2005 describes only the marine portion of the JMA temperature record.

Response: Noted thanks. This reference will be corrected.

Comment: Page 2, line 15 – The latest version of HadCRUT, HadCRUT5, has the following reference. Note the date for the final published version as 2021, not 2020:
Response: Thanks for highlighting. This will be corrected.

Comment: Page 2, line 15 – The 2013 paper for Berkeley Earth only described the land data. The recent paper describing the merged land-ocean can be cited as:

Response: Noted thanks. This reference will be corrected.

Comment: Page 2, line 19 – The GISS data set has long only used satellite nightlight data for bias adjustment of urban areas. It does not use satellite derived temperature information. The current version of MLOST does not use satellite data. The statements here likely refer to the ERSST3b sea surface temperature data set, which used satellite data and was once the marine data source for these data sets. The current version, ERSST5, does not use satellite data.

Response: Noted thanks. This line will be modified accordingly.

Comment: Page 2, line 26 – HadCRUT4 is not interpolated but the recently publish HadCRUT5 is. The JMA data set's oceans are interpolated.

Response: We will clarify this line.

Comment: Page 2, line 30 – MLOST is not based on linear interpolation. It’s a combination of “low frequency” spatial running average and a “high frequency” reduced space analysis using a method called Empirical Orthogonal Teleconnections.

Response: Noted thanks. This line will be clarified.

Comment: Page 2 line 31 – GISS uses linear distance weighting, not inverse linear distance weighting. No inverse involved (see Equation 1 of Lenssen et al., 2019 for the equation, or section 2 of Hansen et al., 2010 for a description). The linear distance weighting is correctly stated in the following sentence on line 32.

Response: We will rephrase this. By linear inverse distance weighting we mean that the weight of each sample point decreases linearly from unity to zero. It does not explicitly include inverse. This interpolation scheme computes estimates by weighting the sample points closer to the prediction location greater than those farther away without considering the degree of autocorrelation for those distances.

Comment: Page 2, line 34 – A reference is needed here. Does this use of kriging refer to the JMA COBE SST data set's use of optimum interpolation?

Response: Yes. We will add the reference as well.

Comment: Page 3, line 1 – Cowtan and Robert (2014) should be Cowtan and Way (2014) (i.e. the author’s name is Robert Way). Repeated again in other references to this paper (e.g. on line 3).

Response: Thanks. This will be corrected throughout the paper.

Comment: Page 3, line 4 – I would argue that these methods do not ignore variations at multiple length scales. For kriging/Gaussian process regression, the ability to represent multiple length scales is dependent on the covariance function used (which can be extremely flexible if constructed to be). The reconstruction method in NOAAGlobalTemp also represents multiple length scales in its own way though a reduced space decomposition. I assume that this comment on multiple length scales is alluding to MLRK, which explicitly represents multiple length scales as a sum of covariance functions. The
distinction here is perhaps in MRLK flexibly to fit covariance structures with multiple scales without necessarily defining those structures in advance?

**Response:** We will modify and tone down this statement.

**Comment:** Page 3, line 5 – The new HadCRUT5 data set include a conditional simulation step to encode analysis uncertainties into an ensemble. Other data sets provided uncertainty estimates in their interpolation by other means but not through simulation.

**Response:** Noted thanks we will incorporate this.

**Comment:** Paragraph at page 3, line 15 - It is great to see the hyperparameter uncertainties and conditional simulation included. It does not appear that all components of the HadCRUT4 uncertainty model have been used though. In particular, those associated with biased observations for individual ships encoded into the HadCRUT4 error covariance matrices and per-grid-cell uncertainty estimates, not included in the ensemble, are not used. Instead the model seems to assume i.i.d. errors for each observed grid cell, with a stationary measurement error variance across all locations, estimated each month.

**Response:** We will explicitly highlight that only the uncertainties encoded in HadCRUT4 ensemble members are included in the study. We will hopefully consider gridded error estimates in our future work.

**Comment:** Page 3, line 27 – Should this comment on sparsity in covariance matrices refer instead to sparsity in inverse covariance matrices?

**Response:** Yes. Calculating the inverse is easier for sparse matrices as compared to those of dense matrices.

**Comment:** Page 4, line 19 – I know that I can refer to Nycha et al. (2015) to understand the function of the lambda parameter. The average reader will not know this. A reference to Nycha et al. (2015) would be appropriate here. Is “aw” one term or two? I don’t understand what this parameter does and I can’t find it in Nycha et al.

**Response:** We will add reference and clarify “aw” which is one term.

**Comment:** Page 4, line 21 – Typo? “The smoothness parameter lambda influence throughout the calculation”. What does is it influence?

**Response:** We will modify this line.

**Comment:** Page 5, line 12 – Are these semivariances defined at the observation locations. Is there any binning etc. to compensate for biases sampling of e.g. short ranges when computing the empirical semivariogram?

**Response:** These are empirical semi variances that are computed for observed spatial locations. All the standard rules are observed while computing the semivariogram. For example, the number of point pairs at each lag distance (or in each bin) is at least 30. Semivariogram is computed up to half of the maximum distance between the points over the whole spatial domain.

**Comment:** Page 5, line 4 – d and rho are not defined in this paper.

**Response:** We will define these terms in the paper.

**Comment:** Page 5, line 10 – version 4.5.0.0 but it’s appropriate for comparisons with Ilyas
et al. (2017) if it is the same version as used there.

Response: This paper and Ilyas et al. (2017) are based on the same version 4.5.0.0.

Comment: Page 7, line 6 – This Section 3.2 is essentially a recap of Morice et al (2012), with a heavy focus on the land data. Only the large-scale bias terms are discussed here and not the measurement and grid sampling uncertainty components. These are particularly important for marine regions as ship/buoy movement leads to spatially correlated error, which should be mentioned here. HadCRUT4 does not include these in the ensemble, but instead as additional spatial error covariance matrices. It seems that these have not been used in this paper.

Response: We will discuss measurement and gridded sampling error estimates as well in this section. However, it is important to note that these are not being used in this paper.

Comment: Page 7, line 3 – The sentence should not begin with “So”.

Response: This line will be modified.

Comment: Page 7, line 24 – It could be noted that the error model here represents the effects of potential residual biases when using station records that have been screened for urbanisation.

Response: This will be added.

Comment: Page 7, line 28 - Sampling distributions for the HadCRUT4 ensemble are described in Morice et al. (2012). It would be sufficient here to refer to that study for the ensemble sampling methods rather than repeating it here and elsewhere in Section 3.2. The output of Morice et al. (2012) is used in this paper rather than reimplementation/modification of their methods so these methodological details are not core to this study.

Response: Sure, we will exclude excessive sampling details and cite Morice et al. (2012) for details.

Comment: Page 8, line 4 – Again, sampling distributions used in the construction of the input datasets used could be replaced with a reference to Morice et al. (2012) as they are not critical to the new work undertaken in this study.

Response: Noted. We will exclude sampling details from here too.

Comment: Page 8, line 20 – Estimation of hyperparameters for each individual choice is an important design choice. There is no discussion of variation of the parameter estimates seasonally or in time later in the paper. It would be interesting to see this.

Response: We will discuss how the parameter estimates vary across time.

Comment: Page 8, line 24 – this is 10 hyperparameter sample draws for each month, yes?

Response: That’s true.

Comment: Page 28, line 28 – do these marginal variances have interpretable units? Are these °C^2? It’s interesting if the marginal variance has little affect on uncertainty as it controls the variance of the process in interpolated regions and the relative importance of each spatial scale. Or is the uncertainty in process variances somehow pushed into lambda in the model’s parameterisation?
The next sentence says that this parameter is computed from a single field. Is there no seasonal variability in marginal spatial variance? Perhaps some comment on how the parameter should be interpreted would be helpful to explain why February 1988 is representative of the whole data set.

**Response:** We estimated the marginal spatial variance for the field that has the maximum information. This slightly varies across time. These details will be added.

**Comment:** Page 9, line 2 – Use of the field with minimum coverage seems a strange choice rather than using a well sampled period. Is this because of computational cost limitations?

**Response:** Not really. We intend to show the coverage error estimates of the least sampled spatial field. Therefore, the posteriors are being shown for the corresponding spatial field to develop a link.

**Comment:** Figure 2 - It would be interesting to see the likelihood-based estimate here too. This would help to understand what's happening in Figure 3 in comparisons between ABC and likelihood-based fields. This figure would benefit from some accompanying discussion of the effects of the parameters on the fitted fields and how the parameter estimates differ from the likelihood-based parameters.

For example, Nychka et al (2015) indicates that lambda =noise variance / process variance. High values of lambda would give a process with low variance and large measurement noise. Would this result in e.g. lower variance fields than a likelihood based estimate of a lower lambda?

**Response:** We will mention maximum likelihood estimates and discuss them with ABC posteriors.

**Comment:** The aw parameter is interesting here. There's a lot of weight right at the edge of the prior distribution. Is the distribution being truncated by the choice of prior?

**Response:** Not really. The choice of prior was guided by Nychka et al. (2015).

**Comment:** Page 9, line 11 – Some discussion of how parameters compare between ABC and likelihood methods would again be useful here. What is it about the sampled parameter values that leads to the differences?

**Response:** Sure, we will add discussion on ABC and maximum likelihood estimates.

**Comment:** Page 10, line 2 – The lower uncertainty in unobserved grid locations could explain why This could be the reason that global average statistics do not appear to be much affected. It seems like the ABC ensemble is leaning towards a model with greater observational noise variance and lesser process variance. This would explain the smoother fields in the figure 2 (a)-(b) comparison. Is this correct?

**Response:** This can be a reason.

**Comment:** Figure 3 – Are these fields the ensemble means/medians? This is not stated. Are the results for this month representative of other months in terms of parameter estimates and uncertainty estimates? It would be useful to see how they compare for better sampled periods or other times of year. The plot needs units (°C?).

**Response:** The field is ensemble median. The field represents a worst-case scenario (i.e. least spatial coverage). Parameter estimates and uncertainties vary across months. We
will mention these details and add units.

Comment: Figure 4 – How is uncertainty defined here? Is this the full ensemble spread? What is the statistic being shown? What are the units?

Response: Figure 4a represents the gridded standard error in °C associated with the gridded predictions made in Figure 3a. The predictions are for the median ensemble member.

Comment: Page 12, line 5 – I think that this says that samples are drawn from the conditional normal distribution, with each HadCRUT4 ensemble member having 10 hyperparameter samples, and each of those having 100 samples from the conditional normal. However, the wording of “namely the variogram based ABC posteriors of autoregressive weights and smoothing parameter” does not include the conditional normal sampling.

Response: That’s true.

Comment: Figure 5 – Is there a grey line plotted for the ensemble median of the old ensemble? If so then I can't see it. It would be helpful to include to show any differences (or lack thereof) in the mean/median with the hyperparameter sampling. Figure needs units (°C?).

Response: Noted. We will try to improve this figure.

Comment: Page 14, line 5 – It’s not clear that the uncertainties are always larger for ABC. They seem comparable or slightly skewed relative to Ilyas et al 2017. Differences often appear to be around the scale of the apparent rounding resolution in the plot. It looks like the uncertainty range is often narrower for the new ensemble.

Is it guaranteed that the uncertainty estimates would be wider using ABC? Could lower uncertainties be possible if the hyperparameter distribution samples a region of the parameter space that leads to smaller process variance, and hence smaller coverage uncertainty estimates?

Response: It is not guaranteed. We will improve this line and add details.

Comment: Page 14, paragraphs at lines 13 and 24 – This discussion of Latin hypercube sampling is a strange and abrupt way to end the paper. It would better be placed earlier in the method/results. This sampling is good to see though as a 100,000-member gridded dataset would be rather unwieldy to use.

Response: Noted. We will discuss Latin hypercube sampling earlier in the paper.

Comment: Page 15, line 1 - No conclusions section? See main point 6.

Response: We will add a conclusion section.

Comment: Figure 6 – The axes need labels/units. It could be useful to see the resulting sampling for other locations too.

Response: We will improve this figure and add more locations in the appendix maybe.