Three novel copula-based bias correction methods for daily ECMWF air temperature data

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Abstract. Data retrieved from global weather forecast systems are typically biased with respect to measurements at local weather stations. This paper presents three copula-based methods for bias correction of daily air temperature data derived from the European Centre for Medium-range Weather Forecasts (ECMWF). The aim of these methods ~~The aim~~ is to predict conditional copula quantiles at different unvisited locations, accounting for the temporal variability of copula's parameter and assuming spatial stationarity of the underlying random field. The ~~three~~ new methods are~~:~~ bivariate copula quantile mapping (~~types~~ BCQM-I and BCQM-II), and a quantile search (QS). In the BCQM methods, quantile mapping is performed between two bivariate copulas. The difference between BCQM-I and BCQM-II is the choice for a particular covariate. The QS method allows one to generate a random variable and to re-estimate the bivariate copula minimizing the error between the true marginal quantile and the marginal quantile estimated by the BCQM methods. These are compared with commonly applied methods, using eleven years data from an agricultural area in the Qazvin Plain in Iran. This area contains ~~containing five~~ eight weather stations. Cross-validation is carried out to assess ~~the~~ performance of the new methods. The study shows that ~~the new methods~~these are able to predict the conditional quantiles at unvisited locations, improve the higher order moments of marginal distributions, and take the spatial variabilities of the bias-corrected variable into account. The study~~It~~ further illustrates h~~ow a choice of the bias correction method affects the bias-corrected variable and highlights~~ both theoretical and practical issues of the methods like how they affect the bias corrected variable. We conclude that the three new methods improve local refinement of weather data, in particular if a low number of observations is available.

1 Introduction

Weather stations are often sparse and usually located at irregular positions. If their data are used for crop growth simulations, then their results at unvisited locations are likely to be uncertain. A solution to this problem is to use weather data from a weather forecast system at ~~each~~ those locations. A modern and reliable weather forecast system is commonly composed of dynamical models, data assimilation methods and a product delivery system (Persson 2013). The coarse resolution of models, mutual dependence of weather parameters, and variability of these parameters in space and time are major sources of uncertainties in a weather forecasts~~, however, result in system uncertainties of the obtained weather data~~ (Dee et al. 2011;

Durai and Bhradwaj 2014). ~~The uncertainties~~ Uncertainties propagate ~~as they~~ if those are further applied, e.g. in hydrological models that increasingly use such data as input. ~~This requires~~ Hence, the data have to be corrected before being used.

Bias is defined as the systematic underestimation or overestimation of a global weather forecast system with respect to local measurements from weather stations (Persson 2013; Mao et al. 2015). Due to the coarse spatial resolution of ~~ECMWF~~ weather forecasted gridded data, there is an apparent mismatch between measurements obtained from weather stations and weather forecast data. In ~~the~~ this study ~~area~~, ~~however,~~ unvisited locations are ~~many~~ grid points which do not contain an observation due to the relatively low number of weather stations in the study area. In order to obtain unbiased values, a bias correction method should be applied for these grid points before using the weather forecast data.

Various bias correction methods have been proposed in the literature: linear-scaling factor methods (Lenderink et al. 2007), nonlinear methods (Lafon et al. 2013), and quantile mapping methods (Ines and Hansen 2006). Although ~~These methods~~quantile mapping methods are able to correct for bias in the mean and standard deviation, ~~but they do not consider~~none of them can robustly correct other moments of a probability distribution (Lafon et al. 2013). Currently, the ~~bivariate~~ Gamma and empirical distributions are specifically used for bias correction of precipitation data (Lafon et al. 2013; Kum et al. 2014) and the Gaussian distribution for bias correction of temperature data (Teutschbein and Seibert 2012). A limitation of this approach is that the same distribution~~s~~ families are used to estimate both the marginal and the multivariate distributions (Genest and Favre 2007). ~~For this reason, we turn to copulas.~~

A copula joins a multivariate distribution to its univariate marginals, based upon Sklar's theorem (Sklar 1973; Nelsen 2006). Copulas describe the complex dependence structure between variables independently from the marginal distributions (Gräler and Pebesma 2011). Recently, copula-based methods have been developed for deriving bias-corrected weather data (Mao et al. 2015). ~~Here,~~In copula-based ~~a~~methods, a conditional distribution describes the dependence structure between weather forecast data and measurements at weather stations. Their estimated quantiles are transformed into bias-corrected weather data.

A bias correction method proposed by Laux et al. (2011) employed ~~a~~ bivariate conditional copulas ~~distribution for~~ to model dependence between the daily precipitation time series retrieved from a regional climate model and observations at three locations where data is available. In their method, however, a bivariate copula is fitted to daily time series at one location, ignoring the temporal variability of copula parameter as well as spatial dependency. In addition, the fitting is required to remove autocorrelation and heteroscedasticity which may exist in the time series (Laux et al. 2011). Mao et al. (2015) investigated daily precipitation data and showed that a copula-based bias correction performs better than quantile mapping. Vogl et al. (2012) proposed the "Multiple Theta" and the "Maximum Theta" approaches for bias correction of rainfall data.

So far, copulas have mainly been applied to precipitation time series retrieved from regional climate models. Observed weather data, in contrast, are provided at various temporal resolutions, whereas bias correction is often assumed to be temporally stationary. This means that they are also valid for future conditions (Teutschbein and Seibert 2012). In addition, in the copula-based methods, conditional quantiles are generated by Monte Carlo simulations and the mean value of the simulations is considered as one solution for the bias-corrected value at an unvisited location (Laux et al. 2011; Vogl et al. 2012; Mao et al. 2015). The drawback of this procedure is further explained in Sect. 2.3.4.

This paper presents three new methods based on ~~the~~ copulas ~~concept~~: bivariate copula quantile mapping (types I and II), and quantile search. In this study, we aim for:~~.~~ ~~The new methods allow for~~

- estimating different conditional quantiles at ~~different~~ all unvisited locations accounting for the temporal variability of the dependence structure. ~~for each time step of a time series. Another aim is to~~
- evaluating~~e~~ ~~these methods~~the ability of these methods to predict the spatial variability of the bias-corrected daily air temperature at unvisited locations. ~~In addition, this paper~~
- comparing~~es~~ the proposed methods with available bias correction methods:~~, which are~~ ~~marginal~~ quantile mapping, expectation predictor and single quantile predictor. ~~The expectation and single quantile predictors are based on the bivariate conditional copula. The aim is to~~
- providing~~e~~ a review and application of these methods for bias correction of the daily air temperature data ~~when~~if a relatively low number of observations are available.

The structure of this paper is as follows. The concept of copulas and the methods of bias correction are presented in Sect. 2. The study area and data are introduced in Sect. 3. The results of bias correction methods for the study area are described in Section 4, followed by the discussion and conclusion in Sect. 5 and Sect. 6.

2 Method

2.1 Copulas

A copula is a multivariate cumulative distribution function that describes the dependence structure between variables. This function is unique if the marginals are continuous functions (Nelsen 2006; Vogl et al. 2012). According to Sklar's theorem, the joint ~~multivariate~~ bivariate distribution H of ~~m~~ two variables Z_i equals a copula C of ~~m~~ two variables ~~u_i~~ U_i as:

$$H(z_1\cancel{,\dots}, z_{\cancel{m}2}) = C(u_1, \cancel{\dots,} u_{\cancel{m}2}), \cancel{.} \tag{1}$$

$$u_i = F_i(z_i), \qquad u_i \in [0,1], \cancel{.} \tag{2}$$

where F_i is the marginal distribution function. A bivariate copula ~~can~~ describes ~~several dependence structures: the spatial dependence structure between two variables at two different locations in space or at two different points in time; the spatio-temporal dependence structure between two variables at different points in time and space; the dependence structure between two variables at one point in time and space. A bivariate conditional copula~~ $\cancel{C_{u_2}^{t}(u_1)}$ ~~is often used to correct for bias by describing~~ the dependence structure between two variables at one point in time and space, where ~~u_1~~ U_1 is treated as "true" variable and ~~u_2~~ U_2 is biased variable. Several copula families have been developed to capture multivariate joint distributions such as ~~the~~ Gaussian, ~~the~~ Student's t, ~~the~~ Clayton, ~~the~~ Gumbel and ~~the~~ Frank ~~families~~ (Nelsen ~~2006~~2003; Joe 1993). These families mainly differ in the way the tail dependence structure is described (Table 1) (Joe 1993; Manner 2007).

2.2 Copula-based bias correction method

The bias at ~~time~~ a single moment t in time and location s in space is defined as the difference between the measurements from weather stations denoted by $z_1^{t,s}$ ~~Z_1~~, and weather forecasts denoted by $z_2^{t,s}$ ~~Z_2~~:

$$\cancel{Z_1} z_1^{t,s} = z_2^{t,s} \cancel{Z_2} + Bias^{t,s} \cancel{Bias}. \qquad (3)$$

The value of the bias is predicted indirectly in copula-based bias correction methods. A bivariate conditional copula C_{u_2} ~~$C^t_{u_2}$~~(u_1) for two variables ~~at one point in space and time~~, denoted by Z_1 and Z_2, is defined as (Nelsen 2006; Gräler 2014):

$$P(Z_1 \le z_1 | Z_2 = z_2) = C_{u_2}(u_1) \cancel{C^t_{u_2}(u_1)} = \cancel{C^t} C(\cancel{u} U_1 \le u_1 | U_2 = u_2) = \frac{\partial \cancel{C^t} C(\cancel{u}U_1, \cancel{u}U_2)}{\partial \cancel{u} U_2} = p_{u_1|u_2},\ C: [0,1]^2 \to [0,1], \cancel{.} \qquad (4)$$

where u_1 and u_2 are empirical marginal quantiles. Throughout, the ~~functions~~ copulas ~~and variable~~ vary over ~~space and~~ time; ~~s refers to a location and t refers to single moment in time~~. We assumed spatial stationarity during each moment in time to estimate $C_{u_2}(u_1)$ ~~C^t~~. This assumption is justified as the dependence structure between the observed and the forecasted variables is studied in a relatively small area and the dependence structures are thus unlikely to change spatially in a non-stationary way.

Empirical marginal ~~distributions~~ quantiles u_1 and u_2 are obtained using the following rank-order-transformation:

$$u_i = \frac{\mathrm{rank}(\cancel{Z} z_i)}{k+1}, \cancel{.}\ (i = 1, 2), \qquad (5)$$

where k denotes the number of available data for Z_i. We denote the transformed variables by $\cancel{u}U_1$ and $\cancel{u}U_2$ for the conditioned and conditioning variables, which are discrete and now approximately uniformly distributed on [0, 1]. Extreme values that possibly exist in the observations, however, ~~are smoothed and hence the extreme values cannot occur~~ are prevented from occurring at unvisited locations after this transformation. To solve for this problem and to obtain a better approximation of the marginal distribution function at unvisited locations, a ~~polynomial~~ spline is fitted to the pairs (z_1, u_1). ~~Yet, this approach is also prone to uncertainty because the polynomial is fitted to a low number of observed values.~~

To further proceed, a bivariate copula is fitted to the marginal quantiles u_1 and u_2 at weather stations for each time step. We use ~~the~~ Student's *t* (Demarta and McNeil 2005), Gaussian, ~~the~~ Clayton, ~~the~~ Gumbel and ~~the~~ Frank families (Joe 1993), as these families are sufficiently flexible to capture the dependence structures of the conditioned and conditioning variables pairs (u_1, u_2). Note that a bivariate copula has one parameter, except for ~~the~~ Student's *t* family that has two parameters: one for the correlation and one for the degrees of freedom (Table 1). To estimate the parameters for each family, we apply maximum likelihood estimation (Gräler 2014), using starting values obtained by Kendall's τ, being a measure of association between variables (Nelsen 2006). Based upon their results, the most suitable family is selected according to Akaike's Information Criteria (AIC) (Akaike 1974).

In the literature, several goodness-of-fit tests exist and a good review is provided by Genest et al., 2009. In this paper, except for Student's *t* copulas, goodness-of-fit is tested based on a new Cramér–von Mises statistic $S_n^{(B)}$ proposed by Genest et al., 2009. The $S_n^{(B)}$ is based on Rosenbalt's transform and recommended for best perfomance and consistensy among other tests (Genest et al., 2009). It has practical limitations to implement it for Student's *t* copulas. For Student's *t* copulas, goodness-of-fit is tested based on a the White statistic S_w (Huang and Prokhorov, 2014). It may be instable in estimation of degree of freedom (Schepsmeier, 2016).

In copula-based bias correction methods, when the conditional quantile $p_{u_1|u_2}$ $\cancel{p_{u_1}}$ in Eq. (4) ~~needs to be~~is predicted, it is used to derive the marginal quantile $\hat{u}_1$ as well as the realization of the random variable $\hat{Z}_1$. ~~The solution depends upon the application, i.e. $\hat{Z}_1$ has to be predicted at the unvisited location in space, at the unobserved period in time or both. Incorporating temporal/spatial information or spatio-temporal information of available data to predict the conditional quantile then likely affects selection of a suitable method.~~

2.3 Realization of random variable $\hat{Z}_1$

The purpose of ~~the~~ bias correction ~~method~~ is to predict the bias-corrected values $\hat{z}_1$ at unvisited locations. This section describes briefly available methods ~~to obtain the realizations of the bias-corrected variable $\hat{Z}_1$~~ which are quantile mapping, expectation predictor, marginal transformation based on a single quantile, and simulation of conditional quantile. In addition, the newly developed methods are explained and compareds ~~them~~ to the quantile mapping and expectation predictor ~~three newly developed methods~~. We utilized the concept of bivariate conditional copula to develop new methods for bias correction, as bivariate copulas are well understood and easy to estimate. The flexibility in the determining of the conditional quantiles makes the newly developed methods appealing for spatial variabilities at unvisited locations when low number of observations are available. The combination of covariates in a Vine copula (Aas et al., 2009) might improve the bias correction, but is out of scope of this paper.

2.3.1 ~~Marginal q~~Quantile mapping

A comprehensive study carried out by Teutschbein and Seibert (2012) showed that ~~the~~ quantile mapping (QM) ~~method~~ performs best among the classical bias correction methods and it can easily be implemented. It can reduce bias in the first two moments of a probability distribution. It is, however, sensitive to the number of quantile divisions when using an empirical probability distribution. For this method, several names can be found in the literature, such as probability mapping, CDF matching, quantile-quantile mapping. ~~Here, we call this method as marginal quantile mapping (MQM) to specify the type of cumulative distribution function in the mapping and compare it to the copula-based bias correction methods.~~ In QM ~~the marginal quantile mapping~~, a single value $\hat{z}_1$as a realization of the random variable $\hat{Z}_1$ is obtained as:

$$\hat{z}_1 = (F_1^t)^{-1}\left(F_2^t(z_2)\right), \cancel{.} \tag{6}$$

where F_1 and F_2 are marginal distribution function of the measurements from weather stations and weather forecasts, respectively. The marginal distribution functions are spatially stationary during each moment t in time. The idea of ~~M~~QM is that there is a perfect dependence between variables ~~u~~U_1 and ~~u~~U_2. This underlying assumption, however, is hard to be fulfilled, due to the complexity of the dependence structure between measurements and forecasted data.

2.3.2 Expectation predictor

The conditional expectation is the optimal predictor, in the sense that it minimize the Bayes risk (Cressie 1993). It can be either linear or nonlinear in Z_1. A single value $\hat{z}_{1(mean)}$ as a realization of the ~~random~~ biased-corrected variable $\hat{Z}_1$ $\cancel{\hat{Z}_1^{t,s}}$ is obtained using the conditional expectation (Bárdossy and Li 2008):

$$\hat{z}_{1(mean)} = E[z_1|\, z_2] = \int_{z1} Z_1 . f^{\cancel{t}}(Z_1 \le z_1|Z_2 = z_2)\cancel{(Z_1|Z_2)}dZ_1 = \int_0^1 (F_1^{\cancel{t}})^{-1}(\cancel{u}U_1).\cancel{c^t}c(\cancel{u}U_1 \le u_1|\cancel{u}U_2 = u_2)d\cancel{u}U_1 \ ,\cancel{.} \quad (7)$$

~~Where~~where $\hat{z}_{1(mean)}$ is the mean value of the variable $\hat{Z}_1$, $E[|]$ is conditional expectation operator, Z_1 and Z_2 are measurements and forecasted variables, respectively, u_1 and u_2 are marginal quantiles of the variables Z_1 and Z_2, F_1 is marginal distribution function of the measurements from weather stations, and $\cancel{c^t}c$ is the conditional copula density function. The marginal distribution functions and copulas are spatially stationary during each moment t in time. In the case of constructing bivariate copulas, it can be shown that (see Appendix 1):

$$\cancel{c^t}c(U_1 \le u_1|U_2 = u_2)\cancel{(u_1|u_2)} = \cancel{c^t}c(u_1, u_2) = \frac{\partial^2 \cancel{C^t}C(\cancel{u}U_1, \cancel{u}U_2)}{\partial \cancel{u}U_1 \partial \cancel{u}U_2}. \quad (8)$$

The expectation predictor (EP) is mostly used for copulas to predict the value at an unvisited location in space (Bárdossy and Li 2008) or to predict the value at an unvisited location in space and time (Gräler and Pebesma 2011) using a large number of observations. In copula-based bias correction methods, however, spatial variability around unvisited locations faces the smoothing effect of EP. Another drawback concerns the empirical marginal quantiles of the bias-corrected variable $\cancel{\hat{u}_1}$. The conditional expectation is either an increasing or a decreasing function of the conditioning variable if the dependence is positive or negative, respectively. Therefore, after applying EP, the empirical marginal quantiles of the bias-corrected variable $\cancel{\hat{u}_1}$ equals the empirical marginal quantiles of the forecasted variable u_2 or $1 - u_2$ (see Appendix 2).

2.3.3 Marginal transformation based on a single quantile

The conditional quantile $p_{u_1|u_2}\cancel{p_{u_1}}$ in the Eq. (4) specifies that the conditioned variable Z_1 takes a value for a given conditioning variable Z_2. To apply the marginal transformation based on a single quantile method, first, the same quantile p_{u_1} for all locations is used to derive the marginal quantile $\hat{u}_1$, by applying the inverse transformation of the copula $(C^{\cancel{t}})^{-1}$:

$$\hat{u}_1 = (C^{\cancel{t}})^{-1}\left(p_{u_1|u_2}\cancel{p_{u_1}}\middle|U_2 = u_2\right)\cancel{.}, \quad (9)$$

where $\hat{u}_1$ is the predicted marginal quantiles for the bias-corrected variable $\hat{Z}_1$, C is the conditional copula which is spatially stationary during each moment t in time, and u_2 is marginal quantile of the variable Z_2. Then the realization of the biased-corrected ~~random~~ variable $\hat{Z}_1$ is obtained by applying the inverse transformation of its marginal distribution $(F_1^{\cancel{t}})^{-1}$ (Nelsen 2006):

$$\hat{z}_1 = (F_1^{\cancel{t}})^{-1}(\hat{u}_1),\cancel{.} \quad (10)$$

where $\hat{z}_1$ is a single value of the variable $\hat{Z}_1$, and F_1 is marginal distribution function of the measurements from weather stations. As the full conditional distribution of variable of interest is derived, any quantiles $p_{u_1|u_2}\cancel{p_{u_1}}$ can be used for instance, the median value of $\hat{Z}_1$ can be obtained when the quantile $p_{u_1|u_2}$ $\cancel{p_{u_1}}$ is 0.5 for all locations. In this method, the question can be posed which quantile $p_{u_1|u_2}$ $\cancel{p_{u_1}}$best suits for the corrected variable at unvisited locations.

2.3.4 Simulation of conditional quantile

Copula-based bias correction methods are used to obtain the conditional quantiles in order to predict the bias-corrected values at unvisited locations. Simulation of conditional quantiles is one procedure to obtain conditional quantiles. In the simulation of conditional quantiles, realizations of the random variable $\hat{Z}_1$ are obtained by generating independent variates u_2 and $p_{u_1|u_2}$ $\cancel{p_{u_1}}$ uniform on $[0,1]^2$ (Salvadori et al. 2007; Nelsen 2006). These variates are used in Eq. (9) to obtain samples $\hat{u}_1$. These samples are transformed to obtain realizations of the random variable $\hat{Z}_1$ by applying the inverse transformation of the marginal distribution in Eq. (10). The number of samples in the simulations, however, influences the simulation of conditional quantiles. ~~Here~~In the simulation procedure, ~~to obtain a single value for air temperature,~~ a choice for either the mean, or the median or the mode of a simulation provides a single value $\hat{z}_1$ as a realizations of the random variable $\hat{Z}_1$. In the literature, the mean value of the simulations is considered as a single realization (Laux et al. 2011~~; Vogl et al. 2012~~). ~~The number of samples in the simulations, however, influences the simulation of conditional quantiles.~~ When choosing large number of the samples in the simulation and one chooses ~~The~~ either the mean ~~and~~ or the median of the simulations as a single value, the mean or median are equal to the mean ~~and the median~~value as derived from the conditional copulas using expectation predictor ~~methods~~ explained in Sect. 2.3.2 and or the median value as derived using median predictor explained in Sect. 2.3.3 ~~when choosing large number of the samples in the simulation~~ (Mao et al. 2015).

2.3.5 Bivariate copula quantile mapping

This section introduces new bias correction methods (BCQM-I and BCQM-II) including a covariate to consider the spatial structure of the air temperature at unvisited locations. The bivariate copula quantile mapping (BCQM) is a two dimensional quantile mapping method and relies on two bivariate copulas incorporating the dependence of the covariate and the air temperature ~~variables of interest~~ (Verhoest et al. 2015). This method is shown in Figure 1 which can be extended to multi-dimensional quantile mapping using more than one covariate for the air temperature. The difference between BCQM-I and BCQM-II is the choice for a particular covariate.

~~2.3.5.1 BCQM-type I~~

In BCQM-I, one bivariate copula describes dependence structure between forecasted variable and elevation, and other bivariate copula describes dependence structure between observed variable and elevation. The ~~variables R and~~marginal quantiles of elevations $u_{\cancel{R}e}$ are defined as:

$$\cancel{R = \sqrt{(x^s)^2 + (y^s)^2 + (e^s)^2}.} \qquad \cancel{(11)}$$

$$u_{\cancel{R}e} = \frac{\mathrm{rank}(e^s\cancel{R})}{k+1}\cancel{.}, \qquad (\cancel{12}11)$$

~~Where~~ where ~~x^s and y^s are the coordinates (in meters) in the universal transverse Mercator (UTM) coordinate system and~~ e^s is the elevation (in meter) of the unvisited locations. ~~The variable R is treated as a random variable due to uncertainty in positioning and elevation. It indicates effects of land cover and elevation on the air temperature over the study area.~~ The idea of this mapping is to use ~~R~~ elevation and the air temperature to estimate copulas. Then, the conditional quantile $P(Z_2 \le z_2|\cancel{R}E = \cancel{r}e)$ at an unvisited location is used to estimate the conditional quantile $P(Z_1 \le z_1|\cancel{R}E = \cancel{r}e)$ at the same location. For this quantile mapping, two conditional copulas $C^{\cancel{t}}_{u_{\cancel{R}e}}(u_1)$ and $C^{\cancel{t}}_{u_{e\cancel{R}}}(u_2)$ are constructed as:

$$P(Z_1 \le z_1|\cancel{R}E = \cancel{r}e) = C_{u_e}\cancel{C^t_{u_R}}(u_1) = \cancel{C^t}C(U_1 \le u_1|U_e = u_{\cancel{R}e}) = \frac{\partial \cancel{C^t}C(U_e\cancel{u_R},uU_1)}{\partial U_e\cancel{u_R}} = p_{u_1|u_e}, \quad C: [0,1]^2 \to [0,1]\cancel{.}, \qquad (\cancel{13}12)$$

$$P(Z_2 \le z_2|E = e\cancel{R = r}) = C_{u_e}\cancel{C^t_{u_R}}(u_2) = \cancel{C^t}C(U_2 \le u_2|U_e = u_e\cancel{u_R}) = \frac{\partial \cancel{C^t}C(U_e\cancel{u_R},U\cancel{u}_2)}{\partial U_e\cancel{u_R}} = p_{u_2|u_e}, \quad C: [0,1]^2 \to [0,1]\cancel{.},$$

$(\cancel{14}13)$

where u_1 and u_2 are calculated following Eq. (5), $C_{u_e}(u_1)$ and $C_{u_e}(u_2)$ are conditional copulas describing dependence between measurements and elevation, and between forecasted air temperature and elevation, respectively. Substituting the quantiles $p_{u_2|u_e}\cancel{p_{u_2}}$ for $p_{u_1|u_e}\cancel{p_{u_1}}$ into Eq. ($\cancel{13}12$) yields the realization of the random variable $\hat{Z}_1$ as it is explained in Eq. (9) and (10).

~~2.3.5.2 BCQM-type II~~

In BCQM-II, one bivariate copula describes dependence structure between forecasted air temperature and its nearest neighbour, and another copula describes dependence structure between measurements and its nearest neighbour. The idea of the BCQM-~~type~~ II method is to use nearest observed neighbour to an unvisited location to estimate copulas. To select the nearest neighbour to an unvisited location, the distance between two locations is calculated using three dimensional coordinates. Then, the conditional quantile $P\left(Z_2 \le z_2^s \middle| Z_{\cancel{2}1} = z_{\cancel{2}1}^{neigh}\right)$ at an unvisited location is used to estimate the conditional quantile $P\left(Z_1 \le z_1^s \middle| Z_1 = z_1^{neigh}\right)$ at the same location. For this quantile mapping, two bivariate conditional copulas $C_{u_1^{neigh}}\cancel{C^t_{u_1^{neigh}}}(u_1^s)$ and $C_{u_1^{neigh}}\cancel{C^t_{u_2^{neigh}}}(u_2^s)$ are constructed as:

$$P\left(Z_1 \le z_1^s \middle| Z_1 = z_1^{neigh}\right) = C_{u_1^{neigh}}(u_1^s)\cancel{C^t_{u_1^{neigh}}(u_1^s)} = \cancel{C^t}C\left(U_1^s \le u_1^s \middle| U_1^{neigh} = u_1^{neigh}\right) = \frac{\partial \cancel{C^t}C\left(\cancel{u}U_1^{neigh},\cancel{u}U_1^s\right)}{\partial \cancel{u}U_1^{neigh}} =$$

$$p_{u_1|u_1^{neigh}}, C: [0,1]^2 \to [0,1]\cancel{.}, \quad (\cancel{15}14)$$

$$P\left(Z_2 \le z_2^s \middle| Z_{\cancel{2}1} = z_{\cancel{2}1}^{neigh}\right) = C_{u_1^{neigh}}(u_2^s)\ \cancel{C^t_{u_2^{neigh}}(u_2^s)} = \cancel{C^t}C\left(U_2^s \le u_2^s \middle| U_1^{neigh} = u_{\cancel{2}1}^{neigh}\right) = \frac{\partial \cancel{C^t}C\left(\cancel{u}U_{\cancel{2}1}^{neigh},\cancel{u}U_2^s\right)}{\partial \cancel{u}U_{\cancel{2}1}^{neigh}} =$$

$$p_{u_2|u_1^{neigh}}, C: [0,1]^2 \to [0,1]\cancel{.}, \qquad (\cancel{16}15)$$

where u_1 and u_2 are calculated following Eq. (5). The copula $C_{u_1^{neigh}}$ ~~$C^t_{u_i^{neigh}}$~~(u_i^s) is the distribution of the variable of interest at unvisited location, conditioned on its nearest observed neighbour. Substituting the quantiles $p_{u_2|u_1^{neigh}}$~~p_{u_2}~~ for $p_{u_1|u_1^{neigh}}$~~p_{u_1}~~ into Eq.(~~15~~14) yields the realization of the random variable $\hat{Z}_1$ as it is explained in Eq. (9) and (10).

2.3.6 Quantile search

At a single moment t in time and location s in space, there is a conditional quantile $p_{u_1|u_{covariate}}$, where $u_{covariate}$ is marginal quantile of either forecasted variable in EP method or one of the covariates in BCQM methods explained in Sect. 2.3.2 and 2.3.5, respectively. The conditional quantile is estimated using either EP or BCQM methods at an unvisited location. As can be seen in Figure 2, the conditional quantile is used to estimate the marginal quantile $\hat{u}_1$ using Eq. (9). The quantile search (QS) method generates variable U_1 ~~allows the combination of different criteria in estimating~~ and the bivariate copula is re-estimated minimizing the error between the estimated marginal quantile $\hat{u}_1$ and true marginal quantile u_1. ~~the marginal quantiles u_1 at unvisited locations.~~In this way, quantile search steps are as follows:

1) The conditional quantiles $p_{u_1|u_{covariate}}$ are calculated using either EP or BCQM methods for all locations.
2) An initial variable $\widehat{U}_1$ is generated by the search algorithm and the bivariate conditional copula is re-estimated. Then, the conditional quantiles $p_{u_1|u_{covariate}}$ and re-estimated copula are used to estimate the marginal quantile $\hat{u}_1$ using Eq. (9);
3) The mean relative error (MRE) for weather stations is calculated as:

$$MRE = \frac{1}{n}\sum_{s=1}^{s=n}\left(\frac{|u_1^s - \hat{u}_1^s|}{u_1^s}\right), \tag{16}$$

where MRE is the mean relative error and n is number of observations. The search algorithm improve the variable $\widehat{U}_1$ in an iterative process by means of minimizing the MRE.

~~As the marginal quantile u_1 lies in the range [0, 1], it can be estimated using a search algorithm by means of maximizing a fitness function f as:~~

$$\cancel{\hat{p}_{u_1} = \hat{C}_*(\hat{u}_1); \quad *: \{u_2, u_R, u_1^{neigh}\}.} \quad \cancel{(17)}$$

$$\cancel{RE_* = \frac{|\hat{p}_{u_1} - p_{u_1}|}{p_{u_1}}; \; MRE_* = \frac{1}{n}\sum_{s=1}^{s=n}(RE_*^s).} \quad \cancel{(18)}$$

$$\cancel{f(\hat{u}_1) = -\sum w_* \times MRE_*.} \quad \cancel{(19)}$$

~~Here $\hat{p}_{u_1}$ and $\hat{u}_1$ are conditional and marginal quantiles estimated by the quantile search, w_* is arbitrary weight set equal to 0.33 in this study, MRE_* is the mean relative error, n is number of weather stations, and RE_{u_2} is the relative error between two quantiles of $C^t_{u_2}(u_1)$ and $\hat{C}^t_{u_2}(\hat{u}_1)$ as explained in the Sect. 2.2. RE_{u_R} is the relative error between two quantiles of $C^t_{u_R}(u_1)$ and $\hat{C}^t_{u_R}(\hat{u}_1)$ as explained in the Sect. 2.3.5.1. $RE_{u_1^{neigh}}$ is the relative error between two quantiles of $C^t_{u_1^{neigh}}(u_1)$~~

~~and $\hat{C}^t_{u_1^{neigh}}(\hat{u}_1)$ as explained in the Sect. 2.3.5.2. The MRE ensures that the prediction's errors are minimized at the weather stations. The RE_* allows us to ensure that dependence structure of the observed and forecasted variables as well as the observed variable and covariates are considered in the finding the marginal quantile. Values of the fitness function $f(\hat{u}_1)$ are calculated using initial random values for $\hat{u}_1$ and the search algorithm improve the quantile $\hat{u}_1$ in an iterative process. Therefore, the fitness values should well represent the estimation errors and the dependence structures at unvisited locations.~~

A realization of the random variable $\hat{Z}_1^{t,s}$ is obtained using a marginal transformation in Eq. (10) based on the estimated quantiles at unvisited locations in Eq. (9). In this study, a linear combination of MREs are used which are calculated based on three bivariate copulas $C_{u_2}(u_1)$, $C_{u_e}(u_1)$ and $C_{u_1^{neigh}}(u_1^s)$ as explained in Sect. 2.3.2 and 2.3.5, respectively. There are several methods that lead to the minimization of the error (Burke and Kendall, 2014). ~~Here~~In this study, we applied a genetic algorithm for doing the search. Details on this algorithm can be found in the literature (Sastry et al., 2013) and are beyond the scope of this paper. The sample code to implement in R, however, is given in the appendix 3.

2.4 Evaluation of the copula-based bias correction methods

The newly developed methods are applied to each time step of the air temperature time series. These time steps represent different bias and dependencies structures between the observed and forecasted variables. The observations from weather stations are used for cross-validation to quantify the robustness of the each method (Lafon et al. 2013). To this end, one observation $z_1^{s,t}$ is removed from the dataset and the bias-corrected value $\hat{z}_1^{s,t}$ is calculated for this point using the reminder of the stations. This method is repeated for all stations. For each observation assigned to the one location *s* and time *t*, that is not included in the bias correction process, the absolute error (AE) is determined, using:

$$AE^{s,t} = \left|\hat{z}_1^{s,t} - z_1^{s,t}\right|. \qquad (\cancel{20}17)$$

The spatial mean absolute error (SMAE) is calculated at each weather station as:

$$SMAE^s = \frac{1}{T}\sum_{t=1}^{t=T}(AE^{s,t}), \qquad (\cancel{21}18)$$

where T is the number of time steps in time series. To compare the five bias correction methods based on the SMAE, an error score (ES) is calculated ~~based on the SMAE for each method~~ at each weather station (Durai and Bhradwaj 2014). ~~A minimum value of~~ The smallest ~~the error score~~ ES indicates for the ~~minimum~~ smallest SMAE. The error measures do not provide any spatial information of the bias-corrected variable. The idea behind the SMAE was to provide criteria when one can compare different methods. A low number of observations can hinder a deeper analysis. The overall prediction quality depends on a good model of the copula, a good fit of the marginal distributions as well as the number of the observations.

In addition, the correlation coefficient r ~~(CC)~~ between observed and bias-corrected values is calculated at each weather station as:

$$\cancel{CC}r^s = \frac{cov\{Z_1^s, \hat{Z}_1^s\}}{\sigma_{Z_1^s}\sigma_{\hat{Z}_1^s}}; \quad Z_1^s = \left\{z_1^{s,1}, z_1^{s,2}, \ldots, z_1^{s,T}\right\}, \quad \hat{Z}_1^s = \left\{\hat{z}_1^{s,1}, \hat{z}_1^{s,2}, \ldots, \hat{z}_1^{s,T}\right\}, \qquad (\cancel{22}19)$$

~~Where~~ where Z_1^s is the measurement from weather stations ~~observation values~~ ~~and~~, ~~-~~$\hat{Z}_1^s$ is the biased-corrected values obtained by cross-validation, and *T* is the number of time steps in time series.. To compare the five bias correction methods based on *r*, an correlation score (CS) is calculated ~~based on the CC for each method~~ at each weather station. ~~A minimum value of the correlation score~~ The smallest CS indicates for the ~~minimum~~ smallest ~~CC~~*r*.

For investigating the performance of each method to reproduce the ~~high~~ moments of the marginal distribution; mean, standard deviation (as well as coefficient of variation), skewness and kurtosis, the relative error RE^{m_i} is calculated as:

$$RE^{m_i,t} = \frac{|m_i^t - \hat{m}_i^t|}{|m_i^t|}\ ,\cancel{.}\ \ \text{i=1:5,} \qquad (\cancel{23}20)$$

where m_i^t and $\hat{m}_i^t$ are the i^{th} order moment of the marginal distribution calculated using ~~observed~~ measurement ~~values~~ z_1 from weather stations and bias-corrected values $\hat{z}_1$ at ~~time~~ moment *t* in time. The bias-corrected values $\hat{z}_1$ are predicted where correction functions are estimated using the measurement from weather stations ~~observed values~~ and applied to the same locations (Lafon et al. 2013).

The moment mean relative error (MMRE) is calculated at each weather station as:

$$MMRE^{m_i} = \frac{1}{T}\sum_{t=1}^{t=T}(RE^{m_i,t}). \qquad (\cancel{24}21)$$

where *T* is the number of time steps in time series. To compare the five bias correction methods based on the MMRE, an error score (ES) is calculated ~~based on the MMRE for each method and for each moment~~. ~~A minimum value of the error score~~ The smallest ES indicates for the ~~minimum~~ smallest MMRE.

The study was performed in the statistical computing environment and language R using the packages gstat (Pebesma 2004), copula (Kojadinovic and Yan 2010), spcopula (Gräler 2011), VineCopula (Brechmann and Schepsmeier 2013), GA (Scrucca 2012) and the basic packages.

3 Case study

The study area is located between ~~36.30~~ 35.99 and ~~35.99~~ 36.30 latitudes (*N*) and 49.64 and 50.59 longitudes (*E*), with a total area of 3307 km^2 in the Qazvin plain, Iran (Figure 3). This area includes an irrigation network, agricultural fields, dominated by wheat, barely, maize, sugar beet, summer crops and orchards, urban areas, bare soil and natural vegetation. The crop calendar is listed in Table 3. Part of this area has been the pilot for a project aiming at development of a planning and monitoring system to support irrigation management of the Qazvin irrigation network (Sharifi 2013). One of the objectives of this project is to produce daily air temperature map from point measurements ~~and apply it to~~ to be used in crop growth simulations for assessing near-real time crop and irrigation water requirement.

Considering the importance of June in the crop calendar of the study area which is the end of winter crops and beginning of summer crops especially maize, we applied the proposed methods to available dataset of this month. ~~Five~~ Eight weather stations (Table 2) were selected because they had a long range of air temperature measurements available and were well spread over the study area. Minimum and maximum distances between stations are 13 and 78 km, respectively (Figure 3). For all weather stations, the daily minimum and maximum air temperatures are available for the periods ~~1 – 31 March and~~ 1-30 June ~~2014~~2004 to 2014~~, except for the second station on 20 March and 23 June and for the first station on 30 June~~. The quality of measurements and number of missing values differ at each stations (Table 2). Daily air temperature is determined by averaging the minimum and maximum temperatures at each weather stations~~ for each day~~.

We used the operational forecast weather data provided by the European Centre for Medium-Range Weather Forecasts (ECMWF). All ECMWF data are available at 3-hourly and 6-hourly intervals from the ERA-Interim data assimilation system and can be retrieved for a 0.125º lat/lon grid points, corresponding to approximately 13.5 km in the meridional direction (Persson 2013). A sample subset of 3 × 8 grid points is selected for the periods ~~1 – 31 March and~~ 1– 30 June ~~2014~~ 2004 to 2014 which covers the irrigation network (Figure 3).

To analyse the temporal variability of dependence structure which is modelled by copula's parameter, the proposed bias correction methods are applied separately at each day in June 2014. Due to lack of availability of daily air temperature measurements in 2014 over the study area, copulas and marginal distributions are fitted to the eleven years series of the daily air temperature data. ~~Due to the coarse spatial resolution of ECMWF data, there is an apparent mismatch between measurements obtained from weather stations and weather forecast data.~~ To evaluate the proposed methods using cross-validation, ~~To correct for bias in weather forecast data,~~ either an observed value or the average of several observed values corresponds to a single grid point if distance between the station and the grid point is negligible. As shown in the Figure 3, we selected six grid points and corresponding weather stations. For cross-validation, stations number four and seven correspond to grid point four and stations number one and six correspond to grid point twenty four. ~~In the study area, however, many grid points do not contain an observation due to the relatively low number of weather stations. In order to obtain unbiased values, a bias correction method should be applied for these grid points before using the weather forecast data.~~

4 Results

~~This section presents the results, where the observed values are the daily air temperatures at five weather stations, forecasted values are the daily air temperatures obtained from ECMWF, and the bias-corrected values are the results of the bias correction methods (MQM, EP, BCQM-type I, BCQM-type II and QS) for twenty-four grid points during the periods 1 – 31 March and 1 – 30 June 2014.~~

4.1 ~~Outlier and bias~~Bias and moments of marginal distribution

~~The graphical comparison of the observed and the forecasted time series of temperature as shown in Figure 2 identifies both bias and outliers. Abrupt changes in the trend correspond to the outliers (Aggarwal 2013). As can be seen, when there is a drop of the observed air temperature, the forecast system produces outliers. Figure 3 shows the scatterplot between the observed and the forecasted values at each weather station. For all stations, outliers occurred on days 8, 19, 22 and 31 in March. The~~

~~forecasted values are negative on days 22 and 31 in March. Since bias correction was applied separately for each day, there was no need to remove the outliers.~~

~~In addition, the~~ Figure 4 shows the time series of the observed and the forecasted values at each station in June 2014. A graphical inspection ~~comparison in~~ reveals that the daily air temperature is underestimated by ECMWF. The extrapolation of climate information from uncertain measurements and time-varying bias in the ECMWF models ~~and observations~~ are associated with uncertainties in the forecasted data (Dee et al. 2011). The average of bias for all stations and all days equals~~is 3.4°C if the outliers (on the days 8, 19, 22 and 31) are ignored and 4.1~~ 4.5°C in ~~March and~~ June 2014~~, respectively~~. Since there is both spatial and temporal variability in the bias, we were not able to correct for bias at one day and an unvisited location using the average value of bias.

~~Figure 3~~ Figure 5 shows the scatterplot between the observed and the forecasted values at each weather station. As can be seen, the observed air temperature series at stations seven and eight are less correlated with forecasted air temperature series than the other stations and it is expected to affect the cross-validation. Sensors and quality of measurements at these two stations differ from the rest (Table 2).

Figure 6 shows the mean, sample standard deviation, coefficient of variation, skewness and kurtosis for both observed and forecasted values at each day from all weather stations. Since we considered empirical marginal distributions for the observed and forecasted variables, sample moments are calculated using values from all stations at each day during June 2014. This figure shows, in time, clearly visible bias in all moments of the marginal distribution. The daily variability of bias in skewness and kurtosis are higher than other sample moments. Classical bias correction methods are inadequate to improve all order moments of the marginal distribution (Lafon et al. 2013). In addition, spatial variability of the observed values is higher than the forecasted values, based on the coefficients of variation. In Sect. 4.3 below, we investigate how well ~~the moments can be reproduced by~~ the described methods correct for bias in the moment of the distribution and in spatial variability of the bias-corrected values.

4.2 Marginal distributions and copulas

In order to not affect the copula by the estimation of the marginal distribution functions, the empirical marginal quantiles ~~values u_1 and u_2~~ were calculated using the daily air temperature data between 2004 and 2014 ~~the available data~~ for both observed and forecasted air temperature ~~for~~ at each day ~~as mentioned in Sect. 2.2~~. The empirical quantiles ~~, however,~~ are typically limited to the domain defined by the extreme values in the observations. Therefore, ~~A third degree polynomial~~ a spline was fitted to the empirical marginal quantiles of the observed values, ~~u_1 to~~ extending the marginal quantiles towards the unvisited locations as well. The empirical marginal quantiles of the observed values and the fitted ~~polynomials~~ spline as well as empirical marginal quantiles of the forecasted values are presented ~~in for~~ at first day of ~~March and~~ June in Figure 7. It shows a clear gap between the quantiles at lower tails of the empirical distributions which is related to a drop in the air temperature in June 2009. In addition, when there is a drop in the observed air temperature, the ECMWF forecasts result in more underestimations of the forecasted temperature.

For the EP and BCQM-II methods, ~~T~~the bivariate conditional copulas describing dependence between the observed and forecasted variables were fitted to the eleven years series of the air temperature data ~~the empirical marginal quantiles for~~ at each day. In BCQM-I, the bivariate copulas describing dependence between air temperature and elevation were fitted to the eleven years series of the air temperature data and one year elevation data at each day.

Following Section~~.~~ 2.2, five copula families were selected to analyse the dependence structures. These families and their indices are listed in Table 2. In addition, five families were estimated ~~at~~ for each day to assess the temporal variability of copula's parameter and the most suitable family according to AIC was selected ~~according to AIC~~. Table 4 shows the best families indices and Kendall's τ at each day in ~~March and~~ June. ~~As can be seen,~~ Except for the dependence between the forecasted variable and the elevation, suitable families of the dependence between the observed and forecasted variables were non-Gaussian ~~at~~ for most of days in ~~March~~ June and ~~Gaussian at most of days in June and~~ these families covered the range from negative to positive dependences. The dependence between the observed and forecasted variables is described by Gumbel copula for most of days. The negative Gaussian dependences between the forecasted variable and the elevation at all days are related to assumptions in statistical and physical models in the ECMWF forecasts. The selection of families, however, depends upon the number of observations and further research is needed to develop strategies to select them. In addition, as all five families were symmetric, alternative families can be investigated to better describe the dependencies. ~~It must be mentioned that although~~ Although many different families exist allowing for different dependence structures, the computational limitations may be introduced by the calculating the inverse of the conditional copula distribution.

The p-values for the best copula family at each day are listed in Table 4. For all methods, the p-values are higher than 0.3 for most of copulas. For all Student's *t* copulas, p-values obtained using White statistic were approximately one.

4.3 Cross-validation results and the bias-corrected values $\hat{z}_1$

~~Applying the described methods to the same data allowed us to compare the different underlying definitions.~~ Table 5 shows the cross-validation results in terms of the spatial mean absolute error (SMAE) between the observations and the bias-corrected values at each ~~weather stations~~grid points for five bias correction methods and their scores during June 2014. The grid point and corresponding weather station/s are listed in the first two columns of this table. For all methods, errors in grid points four and eleven are higher than the rest as it is also illustrated by a lower correlation between the observed and forecasted variables at weather stations seven and eight (Figure 4). A comparison between the newly developed methods BCQM-~~type~~ I, BCQM-~~type~~ II, QS, the available copula-based method EP and the classical bias correction method ~~M~~QM based upon the error scores (ES) ~~has shown~~shows that QS performed best, followed by BCQM-II, QM ,EP, ~~MQM~~ and BCQM-~~type~~ I.~~, BCQM-type II, in March and June.~~

Table 6 shows the cross-validation results in terms of the correlation coefficient *r* ~~(CC)~~ between the observations and the bias-corrected values at each weather stations for five bias correction methods and their scores during June 2014. *r* values for the new methods denote that the time series of the air temperature were successfully reproduced, although the bias correction methods are separately applied at each day. A comparison ~~between the newly developed methods BCQM-type I, BCQM-type~~

~~II, QS, available copula-based method EP and classical bias correction method MQM~~ among the five bias correction methods based upon the correlation score (CS) ~~has shown~~ shows that QS performed best, followed by EP, ~~BCQM-type II~~, ~~M~~QM, ~~EP~~ BCQM-II and BCQM-~~type~~ I~~,~~ in ~~March and~~ June.

~~The first station has the largest temperature values in both March and June, the second and the fifth stations have the smallest temperature values in March, the third and the fifth stations have the smallest temperature values in June, among the five stations, at most of days. Since for all methods, the empirical marginal distributions were the same, the copulas were unable to capture the extreme values. In addition, the SMAE represents the uncertainties associated to horizontal distances, height differences, differences in land cover and vegetation coverage between the stations and the grid points.~~

Table 7 shows the moment mean relative error (MMRE) ~~between the observations and the bias-corrected values at each weather stations~~ for five bias correction methods and their scores. Since we considered empirical marginal distributions for the observed variable, sample moments were obtained using values from all stations at each day of June 2014. A comparison ~~between the newly developed methods BCQM-type I, BCQM-type II, QS, available copula-based method EP and classical bias correction method MQM~~ among the five bias correction methods based upon the error score (ES) shows ~~has shown~~ that ~~QS~~ new methods performed ~~best,~~better than ~~followed by~~ EP~~,~~ ~~BCQM-type II,~~ and ~~M~~QM~~, and BCQM-type I, in March and June~~.

Figure 6 shows that the observed variable has a higher coefficient of variation than the forecasted variable for instance at days 4, 12 and 30. The spatial variation ~~variabilities~~ and error bars of the bias-corrected variable at ~~some~~ these days for all locations ~~is~~ are shown in ~~and~~ Figure 8~~for March and June, respectively~~. ~~It can be seen that the~~ The spatial ~~variabilities~~ variation obtained by newly developed methods were much higher than those obtained by ~~M~~QM and EP. ~~M~~QM, BCQM-~~type~~ I and BCQM-~~type~~ II were unable to correct for bias at some locations. The smoothing effect of EP ~~can be seen in~~ occurs at all days ~~6 and 13 in~~(Figure 8)~~,~~ as well. ~~The spatial variabilities of bias-corrected values obtained by MQM and EP follow the spatial variabilities of the forecasted values.~~ QS performed better to obtain the spatial variation.~~at the weather stations due to the fitness function in Sect. 2.3.7.~~ How to analyse the spatial variability of the bias-corrected air temperature at unvisited locations is still a challenging question due to low number of observations.

5 Discussion

The dependence structure between the daily air temperature observed by the weather stations and forecasted by ECMWF was studied for bias correction. ~~We utilized the concept of bivariate conditional copula to develop three new methods in the bias correction methods, as bivariate copulas are well understood and easy to estimate. We picked up the idea of the quantile mapping and adapted it to the bivariate conditional copula to develop the new methods BCQM-type I and BCQM-type II that allow estimating different conditional quantiles at different unvisited locations. The flexibility in the determining of the conditional quantiles makes the newly developed methods appealing for spatial variabilities at unvisited locations when low number of observations are available. The estimation of marginal distributions and copulas, however, are affected by the low~~

~~number of observations. In addition, the~~ The new methods ~~quantile search QS was~~ were proposed to find the marginal quantiles ~~that might benefit from a fitness function~~ that does not only take into account the prediction errors, but also the spatial variabilities at the unvisited locations. Furthermore, our proposed methods utilized the flexibility of selecting different families and allowed for temporal variability of dependencies.

We treated the available observations from ~~five~~ eight weather stations as a reference during the identification of bias and during the validation of the results. The horizontal distances, height differences and difference in land cover between the location of a station and the ECMWF grid point is associated with uncertainties. In addition, in the copula-based methods, where we used the AIC to select the suitable family for constructing the dependence between the forecasted and the observed variables, additional uncertainties present because the suitability of family depends on the availability of data and the probabilistic nature of the bias. Furthermore, based on the cross-validation results, the average of the mean absolute errors in all stations and all days appeared to be slightly more than 1°C for all proposed methods. As mentioned in Sect. 3, the bias-corrected air temperature can be used for crop growth simulation as well as determination of crop water requirement. The impact of air temperature variability on crop production is dependent on growing-season temperature and the optimum temperature for photosynthesis and biomass accumulation. Asseng et al. 2011 showed that, depending on the time and temperature, the variation in the average growing-season temperature could cause a significant reduction in wheat grain production. Further studies are necessary to quantify the impact of temperature variability on crop production in the study area.

A practical advantage of the proposed methods is that they are not restricted to remove autocorrelation and heteroscedasticity in time series (Laux et al. 2011) and the time series of the air temperature at each station were successfully reproduced by applying the bias correction separately at each day. Another aspect is the ability of the new methods to reproduce the moments of the marginal distribution of the observed variable. Correction of the higher moments of the distribution is much more sensitive to the choice of the bias correction method, which needs to be investigated more in further studies. In addition, in the proposed methods, the empirical marginal distribution described the statistical properties of daily air temperature without the knowledge of theoretical form of the family's distribution function. Furthermore, fitting a ~~polynomial~~ spline to the empirical marginal quantiles was beneficial to obtain the bias-corrected values at unvisited locations that were not limited to the domain defined by the ~~extreme~~ values in the observations. With respect to the newly described methods, although we applied the methods for correcting the bias, we highlight the potential and the use of the methods for the copula-based downscaling problems, as well. Moreover, the proposed methods have the potential to use the spatio-temporal information of the variable of interest in the bias correction process. The further comparison of the proposed methods and other bias correction methods e.g. triple collocation analysis (Stoffelen 1998) might help to assess the performance of the newly developed methods.

Lack of spatial variability in the available copula-based bias correction methods motivated the research to develop new methods with the aim of estimating different conditional quantiles at different locations. The spatial variability of the air temperature, however, needs additional analysis, as the number of observations is small. ~~Based on the available literature, estimating the confidence intervals is a common task to address the uncertainties in the copula-based methods. The applicability of confidence intervals, however, always depends on the availability of data and the nature of the real world~~

~~problem.~~ In ~~addition, for the~~ BCQM-~~type I and BCQM-type II~~ methods, it is assumed that the ~~associations~~ dependence structure between ~~the pair of~~ the bias-corrected variable and ~~a~~ the covariate should obey the ~~associations~~ dependence structure between ~~the pair of~~ the biased variable and that covariate. For QS method it is assumed that the ~~fitness function~~errors ~~fitted to~~calculated by the observations is an acceptable representation of ~~fitness function~~errors at unvisited locations. In the case the underlying assumptions of these methods are hard to be fulfilled, alternatives are needed.

6 Conclusions

In this paper, we developed three copula-based bias correction methods with the aim of predicting different conditional quantiles at unvisited locations and compared them to available methods. They were applied to correct bias in the daily air temperature forecasts of ECMWF. To evaluate their performance, cross-validation was carried out with the observations from ~~five~~ eight weather stations.

From this study, based on the error measures in Table 5 and 7 and the correlation coefficients in Table 6, we conclude the following:

- The new methods are beneficial for the local refinement of weather data if a low number of observations is available and one is interested in predicting the spatial variabilities of the weather parameter.
- The new methods are advantageous if the bias-corrected variable has to be predicted separately at each time step of the time series.
- Further research should focus on investigating the optimal number of observations for bias correction and on developing validation criteria. In both issues, the spatial variability and the error of the predictions in case of a low number of observations should be included.

Competing interests

The authors declare that they have no conflict of interest.

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Appendix:

1. Conditional copula

In the case of constructing bivariate copulas, it can be shown that:

$c(U \leq u|V = v) = c(U,V) = \frac{\partial^2 C(U,V)}{\partial U \partial V}$,

where $c(U \leq u|V = v)$ is conditional density and $c(U,V)$ is joint density distribution. In copulas, marginals (U,V) are uniformly distributed i.e. *f(U)*=*f(V)*=1, *F(U)*=*U* and *F(V)*=*V*, where *f* and *F* are density and cumulative distribution functions, respectively (Kuipers and Niederreiter, 2012). The conditional cumulative distribution is given as (Nelsen 2006):

$C(U \leq u|V = v) = \frac{\partial C(U,V)}{\partial V}$.

The conditional density distribution is derivative of cumulative distribution to its variable:

$$c(U \leq u|V = v) = \frac{\partial}{\partial U}\left(C(U \leq u|V = v)\right) = \frac{\partial}{\partial U}\left(\frac{\partial C(U,V)}{\partial V}\right) = \frac{\partial^2 C(U,V)}{\partial U \partial V}.$$

In addition, the joint density distribution is derivative of cumulative distribution to its variables: $c(U,V) = \frac{\partial^2 C(U,V)}{\partial U \partial V}$.

2. Conditional expectation

Let $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$ be a set of observations for variables X and Y. If $x_1 > x_2$ and $y_1 > y_2$, the pair is concordant. If $x_1 > x_2$ and $y_1 < y_2$, the pair is discordant. When the number of concordant pairs is more (or less) than discordant pairs, the dependence between X and Y is positive (or negative) (Nelsen 2006) .

The conditional expectation is defined as:

$$E[x|\ y] = \int_x X.f(X \le x|Y = y)dX = \int_0^1 (F)^{-1}(U).c(U \le u|V = v)dU.$$

The conditional expectation is either an increasing or a decreasing function of the conditioning variable i.e. if $x_1 > x_2$ then $E[x_1|\ y] > E[x_2|\ y]$ (Dodds et al., 1990). Therefore, after applying conditional expectation, the empirical marginal quantiles of predicted variable X^{prd}: $\{E[x_1|\ y_1], \ldots, E[x_n|\ y_n]\}$ equals the empirical marginal quantiles of Y i.e. v or $1 - v$ in the case of positive or negative dependence.

3. Genetic algorithm in R

Ga(type = c("real-valued"), fitness = Fitness function,…,min = min_u, max = max_u, popSize =100 ,maxiter = 100, seed=500, parallel = T), where "type" is the type of genetic algorithm to be run depending on the nature of decision variables, the fitness function is any allowable R function which takes as input a vector of length equal to marginal quantiles at unvisited locations representing a potential solution, and returns a numerical value describing its "fitness", min_u and max_u are vector of length equal to the marginal quantiles providing the minimum and maximum of the search space and "popSize" and "maxiter" are the population size and maximum iteration which are selected arbitrary.

Table 1: Five families of copulas ~~are selected to~~ describe the dependence structure between the conditioned and the conditioning variables in this study. A bivariate copula is fitted ~~on~~ to the marginal ~~values~~ quantiles and the most suitable family is selected according to the Akaike Information Criteria (AIC) for each day.

Index	Name	$C_\theta(u,v)$	Property index
1	**Gaussian**	$\emptyset_R\left(\emptyset^{-1}(u),\emptyset^{-1}(v)\right); R=\begin{bmatrix}1 & \theta\\ \theta & 1\end{bmatrix}$	1, 2, 6
2	**Student's *t***	$t_{R,\vartheta}\left(t_\vartheta^{-1}(u),t_\vartheta^{-1}(v)\right);\ R=\begin{bmatrix}1 & \theta\\ \theta & 1\end{bmatrix};\ \vartheta = degree\ of\ freedom$	1, 2, 6
3	**Clayton**	$\left[max\{(u^\theta+v^\theta-1),0\}\right]^{\frac{-1}{\theta}}$	1, 2,4,5,6
4	**Gumbel**	$\exp(-[(-lnu)^\theta+(-lnv)^\theta]^{\frac{1}{\theta}})$	1,2,3,6
5	**Frank**	$\frac{-1}{\theta}\ln(1+\frac{(e^{-\theta u}-1)(e^{-\theta v}-1)}{e^{-\theta}-1})$	1,2,6
1	**Property**	Permutation symmetry	
2		Symmetry about medians	
3		Extreme value Copula	
4		Lower tail dependence	
5		Upper tail dependence	
6		Extendibility to Multivariate Copula	

Table 2: ~~Five~~ Eight weather stations ~~are selected due to the~~ measure air temperature ~~measurements available over the entire~~ in the study area. ~~For all weather stations, minimum~~ Minimum and maximum air temperatures are available for the periods ~~1 – 31 March and~~ 1-30 June ~~2014~~2004 to 2014~~, except for the second station on 20 March and 23 June and for the first station on 30 June~~.

Station ID	Station name	Latitude	Longitude	Elevation(m)	Type	Air temperature measurements	Number of observations during June in 2004 to 2014
1	Abeyk	36.05	50.52	1291	Climatology type1	6 hourly	29
2	Magsal	36.13	50.12	1260	Climatology type1	6 hourly	239
3	Nirougah	36.18	50.25	1318	Climatology type1	6 hourly	270
4	Qazvin	36.25	50.05	1278	Synoptic	3 hourly	330
5	Takestan	36.05	49.65	1283	Synoptic	3 hourly	330
6	Baghkousar	36.06	50.58	1225	Climatology type1	6 hourly	240
7	KampMaskooni	36.27	49.99	1316	Climatology type2	Only minimum and maximum	330
8	DolatAbad	36.16	49.81	1285	Climatology type2	Only minimum and maximum	210

Table 3: Crop data including the start of the growing season and harvesting time in the study area.

Crop	Start of growth	End of growth
Wheat	6 Nov.	5 Jul.
Barley	27 Oct.	12 Jun.
Canola	24 Sept.	14 Jun.
sorghum	22 May	18 sept.
Maize	10 May	12 Oct.
alfalfa	7 Mar.	31 Oct.
Vegetables	4 Apr.	20 Aug.
Grape	21 Apr.	21 Oct.
Fruit	21 Mar.	21 Oct.

~~Table 3:~~ Table 4: The p-value, ~~B~~best ~~fitting~~ family and Kendall's τ at each day for bivariate conditional copulas ~~$C^t_{u_2}(u_1)$~~. The non-Gaussian bivariate copulas dominate the non-spatial dependence structure ~~of the observed and forecasted variables~~ at most of the days in ~~March~~June. The copula family indices are listed in Table1.

	$C_{u_2}(u_1)$			$C_{u_1^{neigh}}(u_2)$			$C_{u_1^{neigh}}(u_1)$			$C_{u_e}(u_2)$			$C_{u_e}(u_1)$		
Day	**p-value**	**Best**	**τ**	**p-value**	**Best**	**τ**	**p-value**	**Best**	**τ**	**p-value**	**Best**	**τ**	**p-value**	**Best**	**τ**
1	0.66	5	0.43	0.46	5	0.56	0.32	5	0.47	0.81	1	-0.08	0.70	3	0.06
2	0.30	3	0.35	1.00	2	0.45	0.30	4	0.44	0.64	1	-0.11	0.60	3	0.08
3	0.74	1	0.29	1.00	2	0.32	0.58	4	0.32	0.68	1	-0.09	0.68	3	0.11
4	1.00	2	0.21	1.00	2	0.31	0.99	2	0.29	0.70	1	-0.12	0.45	4	0.04
5	1.00	2	0.29	1.00	2	0.42	0.95	3	0.35	0.72	1	-0.08	0.56	3	0.07
6	1.00	2	0.35	0.56	5	0.51	0.66	3	0.38	0.83	1	-0.09	0.64	1	0.10
7	0.34	4	0.33	0.17	4	0.37	0.44	5	0.28	0.54	1	-0.12	0.42	3	0.16
8	1.00	2	0.23	1.00	2	0.34	0.98	2	0.17	0.75	1	-0.11	0.65	4	0.03
9	0.26	3	0.44	0.97	2	0.55	0.50	4	0.43	0.70	1	-0.11	0.99	1	0.00
10	0.50	5	0.43	0.36	5	0.54	0.68	4	0.38	0.79	1	-0.16	0.72	1	-0.06
11	0.36	4	0.21	0.17	4	0.25	0.68	4	0.40	0.64	1	-0.14	0.97	4	0.02
12	0.70	1	0.30	0.62	1	0.41	0.99	2	0.20	0.70	1	-0.11	0.38	5	0.16
13	0.50	3	0.30	0.09	3	0.37	0.64	3	0.24	0.79	1	-0.12	0.50	5	0.21
14	0.70	1	0.39	0.46	1	0.52	0.52	3	0.39	0.81	1	-0.10	0.46	5	0.16
15	0.64	4	0.42	0.46	4	0.52	0.99	2	0.31	0.58	1	-0.12	0.77	3	0.08
16	0.99	2	0.27	0.68	4	0.39	0.99	2	0.33	0.83	1	-0.09	0.36	3	0.13
17	0.58	4	0.42	0.40	4	0.51	0.64	4	0.35	0.64	1	-0.12	0.54	3	0.05
18	0.34	5	0.43	0.25	5	0.46	0.46	5	0.38	0.74	1	-0.12	0.66	1	-0.04
19	0.46	5	0.53	0.32	5	0.60	0.42	5	0.43	0.58	1	-0.10	0.46	5	0.03
20	0.97	2	0.46	0.99	2	0.50	0.95	2	0.36	0.72	1	-0.08	0.60	5	0.06
21	0.34	4	0.40	0.83	5	0.40	0.98	2	0.34	0.38	1	-0.15	0.77	1	0.06
22	0.87	4	0.38	0.58	4	0.44	0.83	4	0.28	0.68	1	-0.09	0.70	5	-0.01
23	0.81	4	0.28	0.25	4	0.34	0.56	4	0.25	0.83	1	-0.10	0.54	1	-0.06
24	0.62	5	0.28	0.34	5	0.40	0.62	4	0.36	0.64	1	-0.13	0.52	1	-0.08
25	0.72	4	0.26	0.42	4	0.39	0.79	4	0.38	0.50	1	-0.20	0.68	5	0.11
26	0.72	4	0.32	0.44	5	0.45	1.00	2	0.23	0.83	1	-0.10	0.48	1	0.08
27	0.36	3	0.38	0.34	5	0.58	0.68	4	0.41	0.81	1	-0.11	0.56	1	-0.03
28	0.26	3	0.33	0.21	5	0.44	0.99	2	0.39	0.50	1	-0.20	0.76	4	0.01
29	0.58	5	0.34	0.21	5	0.43	0.89	4	0.46	0.30	5	-0.18	0.87	1	-0.03
30	0.52	3	0.28	1.00	2	0.32	0.64	1	0.36	0.42	5	-0.17	0.62	1	-0.03

~~Table 4:~~ Table 5: Results of ~~The cross~~Cross-validation~~ results~~, ~~which~~ showing the robustness of the proposed bias correction methods. The spatial mean absolute error (SMAE) illustrates the mean absolute errors of all days at each station obtained by the ~~marginal~~ quantile mapping (QM), the expectation predictor (EP), bivariate copula quantile mapping (~~type~~ BCQM-I and BCQM-II), and the quantile search (QS). The last row of the SMAE is the average of SMAE over the study area. To compare the five bias correction methods, an error score (ES) is calculated based on the SMAE for each method at each weather station. A minimum value of the error score indicates for the minimum SMAE. The last row of the ES is the sum of scores for each method and indicates that the quantile search performs ~~better~~ best.

			SMAE						ES				
	Grid Id	**Station ID**	**QM**	**EP**	**BCQM-I**	**BCQM-II**	**QS**	**Station**	**QM**	**EP**	**BCQM-I**	**BCQM-II**	**QS**
June	**24**	**1,6**	1.07	1.48	1.84	0.98	1.26	**1,6**	2	4	5	1	3
	13	**2**	1.13	0.96	1.26	1.24	0.96	**2**	3	1	5	4	2
	14	**3**	1.12	1.33	1.37	1.11	1.13	**3**	2	4	5	1	3
	4	**4,7**	1.89	1.82	1.80	1.73	1.75	**4,7**	5	4	3	1	2
	17	**5**	1.27	1.49	1.36	1.53	1.32	**5**	1	4	3	5	2
	11	**8**	2.72	2.36	2.82	2.57	2.27	**8**	4	2	5	3	1
		Average	1.53	1.57	1.74	1.53	1.45	**Sum**	17	19	26	15	13
~~June~~		~~1~~	~~0.9~~	~~1.7~~	~~1.0~~	~~0.9~~	~~1.4~~	~~1~~	~~2~~	~~5~~	~~3~~	~~1~~	~~4~~
		~~2~~	~~1.1~~	~~1.1~~	~~1.0~~	~~1.5~~	~~1.1~~	~~2~~	~~4~~	~~3~~	~~1~~	~~5~~	~~2~~
		~~3~~	~~1.0~~	~~1.1~~	~~1.0~~	~~1.1~~	~~0.9~~	~~3~~	~~3~~	~~4~~	~~2~~	~~5~~	~~1~~
		~~4~~	~~0.7~~	~~0.8~~	~~0.9~~	~~0.8~~	~~0.7~~	~~4~~	~~2~~	~~3~~	~~5~~	~~4~~	~~1~~
		~~5~~	~~1.3~~	~~1.2~~	~~2.3~~	~~1.4~~	~~1.0~~	~~5~~	~~3~~	~~2~~	~~5~~	~~4~~	~~1~~
		~~Average~~	~~1.0~~	~~1.2~~	~~1.3~~	~~1.1~~	~~1.0~~	~~Sum~~	~~14~~	~~17~~	~~16~~	~~19~~	~~9~~

~~Table 5:~~ Table 6: The correlation coefficient (~~CC~~*r*) between observed and bias-corrected values is calculated at each weather station. The bias-corrected values are obtained by the marginal quantile mapping, the expectation predictor, bivariate copula quantile mapping (type I and II), and the quantile search for all days. To compare the five bias correction methods, a correlation score (CS) is calculated based on ~~the CC~~ *r* for each method at each weather station. A minimum value of the error score indicates for the minimum ~~CC~~*r*. The last row of the ES is the sum of scores for each method and indicates that the quantile search performs ~~better~~ best.

			~~CC~~*r*						CS				
	Grid Id	**Station**	**-QM**	**EP**	**BCQM -I**	**BCQM -II**	**QS**	**Station**	**QM**	**EP**	**BCQM-I**	**BCQM-II**	**QS**
June	**24**	**1,6**	0.91	0.90	0.89	0.91	0.88	**1**	4	3	2	5	1
	13	**2**	0.89	0.91	0.87	0.86	0.90	**2**	3	5	2	1	4
	14	**3**	0.89	0.92	0.88	0.88	0.92	**3**	3	4	1	2	5
	4	**4,7**	0.75	0.82	0.74	0.78	0.79	**4**	2	5	1	3	4
	17	**5**	0.89	0.87	0.89	0.83	0.90	**5**	4	2	3	1	5
	11	**8**	0.20	0.26	0.18	0.25	0.29		2	4	1	3	5
								Sum	18	23	10	15	24
~~June~~		**~~1~~**	~~0.88~~	~~0.81~~	~~0.87~~	~~0.88~~	~~0.87~~	**~~1~~**	~~4~~	~~1~~	~~2~~	~~5~~	~~3~~
		~~2~~	~~0.95~~	~~0.92~~	~~0.94~~	~~0.93~~	~~0.92~~	**~~2~~**	~~5~~	~~1~~	~~4~~	~~3~~	~~2~~
		~~3~~	~~0.92~~	~~0.92~~	~~0.91~~	~~0.90~~	~~0.94~~	**~~3~~**	~~4~~	~~3~~	~~2~~	~~1~~	~~5~~
		~~4~~	~~0.96~~	~~0.96~~	~~0.94~~	~~0.95~~	~~0.97~~	**~~4~~**	~~3~~	~~4~~	~~1~~	~~2~~	~~5~~
		~~5~~	~~0.89~~	~~0.91~~	~~0.78~~	~~0.88~~	~~0.94~~	**~~5~~**	~~3~~	~~4~~	~~1~~	~~2~~	~~5~~
								~~Sum~~	**~~19~~**	**~~13~~**	**~~10~~**	**~~13~~**	**~~20~~**

~~Table 6:~~ Table 7: For investigating the performance of each method to reproduce the high moments of the marginal distribution, the moment mean relative error (MMRE) is calculated. To compare the five bias correction methods, an error score (ES) is calculated based on the MMRE for each method at each weather station. A minimum value of the error score indicates for the minimum MMRE. The last row of the ES is the sum of scores for each method and indicates that ~~the quantile search~~new methods perform ~~s~~ best ~~better~~.

	MMRE						ES					
	Moment	**QM**	**EP**	**BCQM-I**	**BCQM-II**	**QS**	**Moment**	**QM**	**EP**	**BCQM-I**	**BCQM-II**	**QS**
June	**Mean**	0.04	0.03	0.05	0.04	0.02	**Mean**	4	2	5	3	1
	Standard deviation	0.67	0.78	0.56	0.61	0.69	**Standard deviation**	3	5	1	2	4
	Coefficient of variation	0.66	0.78	0.55	0.61	0.69	**Coefficient of variation**	3	5	1	2	4
	Skewness	1.38	1.14	1.21	0.96	1.11	**Skewness**	5	3	4	1	2
	Kurtosis	0.35	0.27	0.36	0.40	0.34	**Kurtosis**	3	1	4	5	2
							Sum	18	16	15	13	13
~~**June**~~	~~**Mean**~~	~~0.01~~	~~0.01~~	~~0.01~~	~~0.02~~	~~0.01~~	~~**Mean**~~	~~3~~	~~1~~	~~4~~	~~5~~	~~2~~
	~~**Standard Deviation**~~	~~0.41~~	~~0.48~~	~~0.69~~	~~0.31~~	~~0.14~~	~~**Standard Deviation**~~	~~3~~	~~4~~	~~5~~	~~2~~	~~1~~
	~~**Skewness**~~	~~1.74~~	~~1.07~~	~~1.73~~	~~1.60~~	~~0.43~~	~~**Skewness**~~	~~5~~	~~2~~	~~4~~	~~3~~	~~1~~
	~~**Kurtosis**~~	~~0.20~~	~~0.21~~	~~0.24~~	~~0.22~~	~~0.08~~	~~**Kurtosis**~~	~~2~~	~~3~~	~~5~~	~~4~~	~~1~~
							~~**Sum**~~	~~**13**~~	~~**10**~~	~~**18**~~	~~**14**~~	~~**5**~~

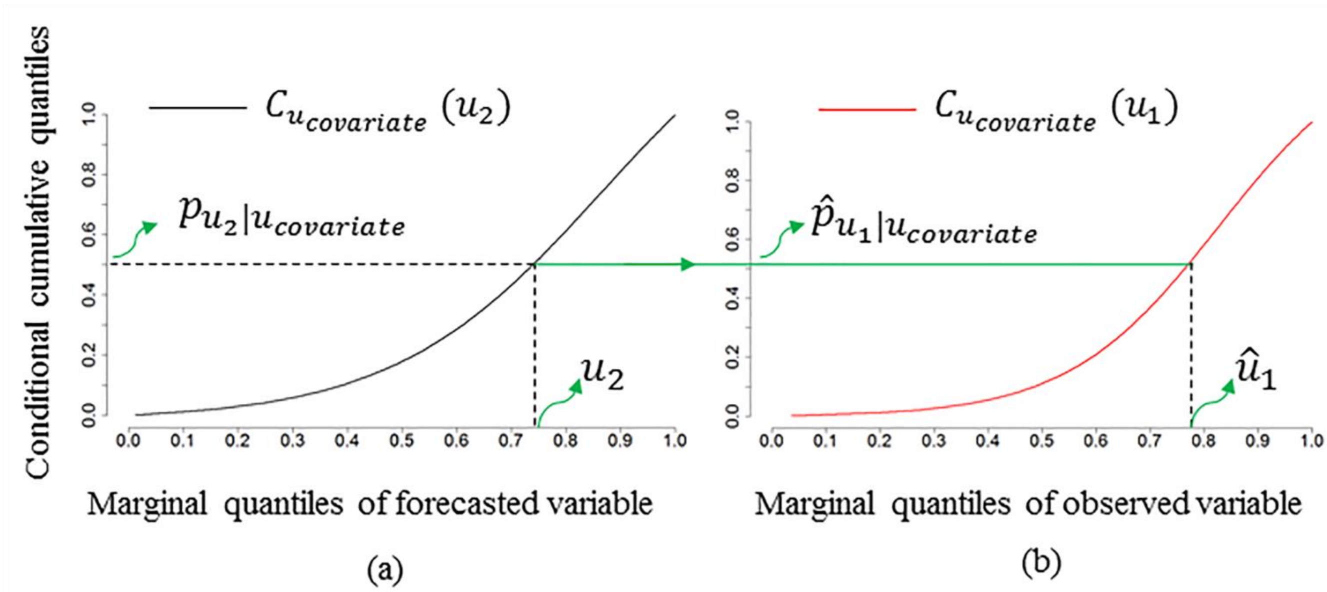


Figure 1: **One bivariate copula describes dependence structure between forecasted variable and covariate (a), and the second describes dependence structure between observed variable and covariate (b). The bivariate copula quantile mapping (BCQM) is substituting the conditional quantile $p_{u_2|u_{covariate}}$ from (a) into $C_{u_{covariate}}(u_1)$ in (b). $u_{\text{covariate}}$ is the marginal of either elevation or nearest neighbour in BCQM-I and BCQM-II, respectively.**

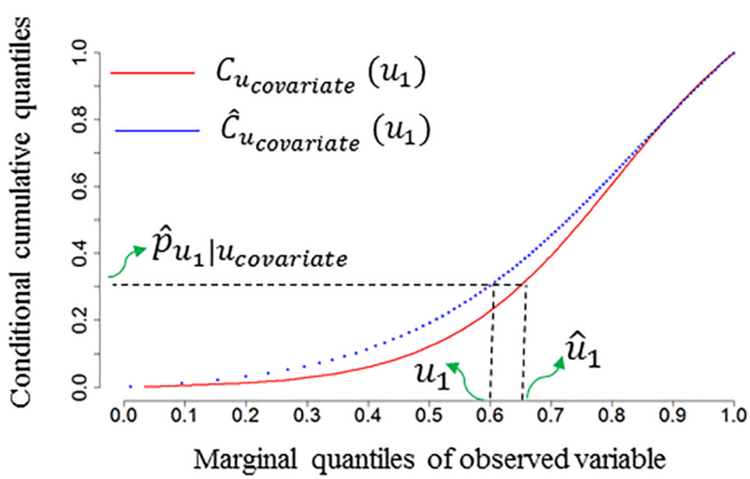
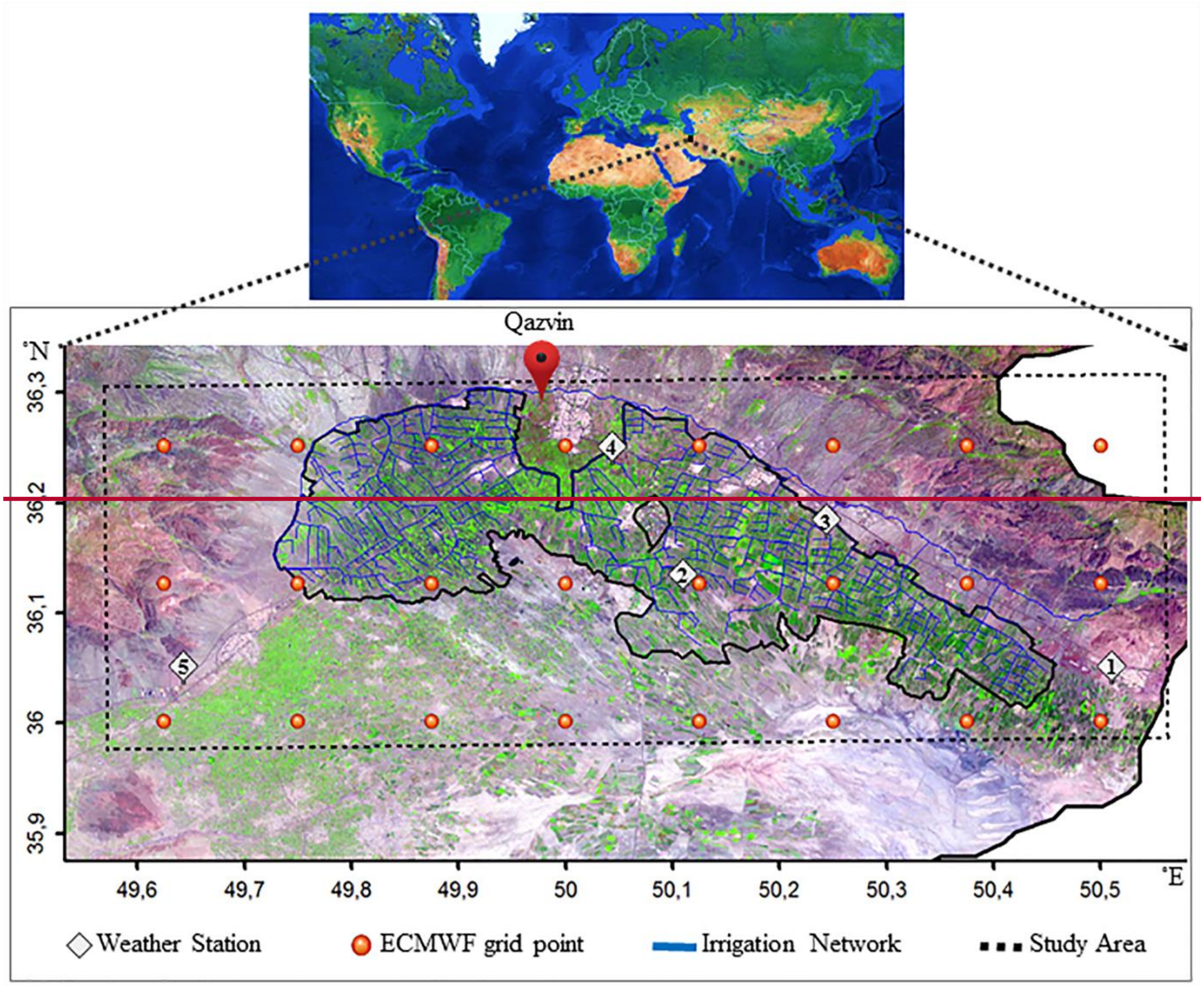


Figure 2: **The conditional quantile $p_{u_1|u_{covariate}}$ and marginal quantile $\hat{u}_1$ of observed variable are estimated using BCQM-I and BCQM-II at an unvisited location. The quantile search (QS) generates a variable U_1 and the bivariate copula $\widehat{C}_{u_{covariate}}(u_1)$ is re-estimated minimizing the error between the estimated marginal quantile $\hat{u}_1$ and the true marginal quantile u_1 at weather stations.**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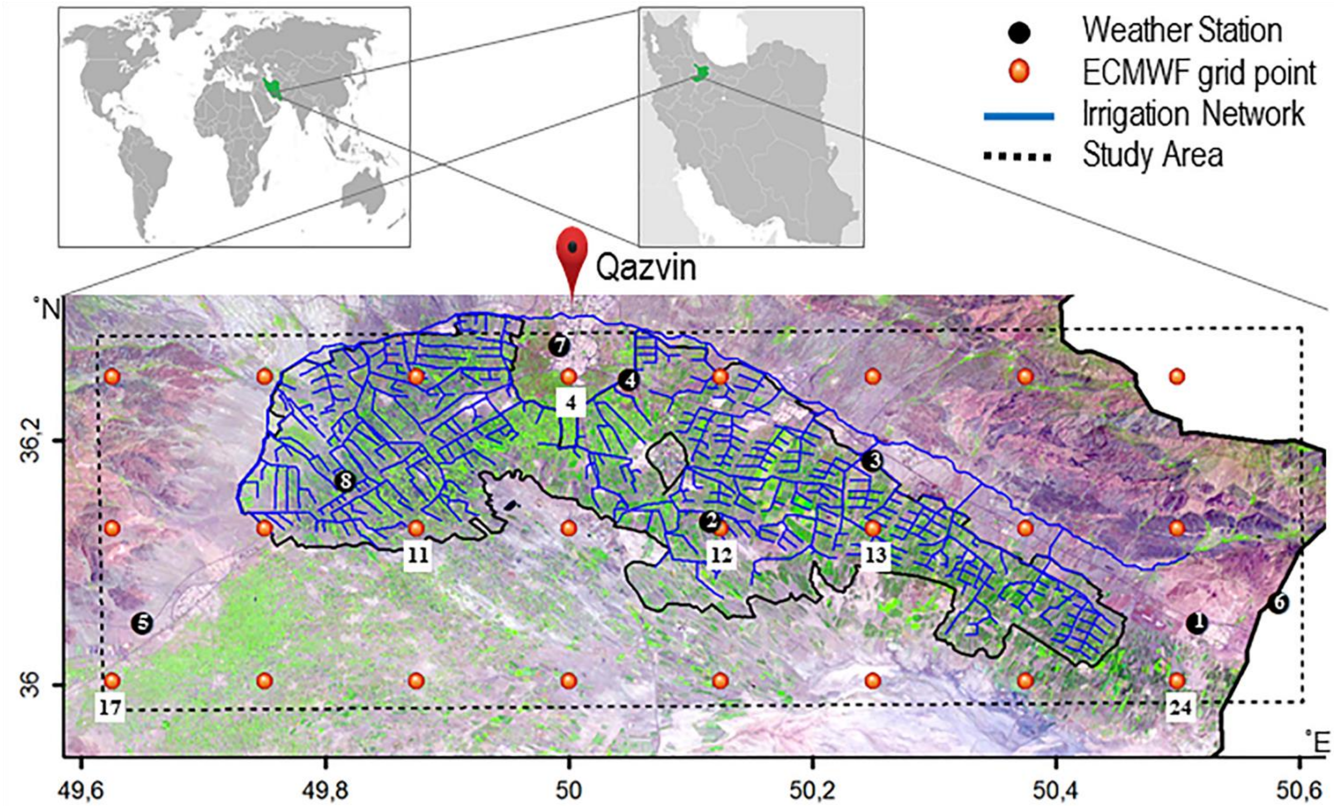
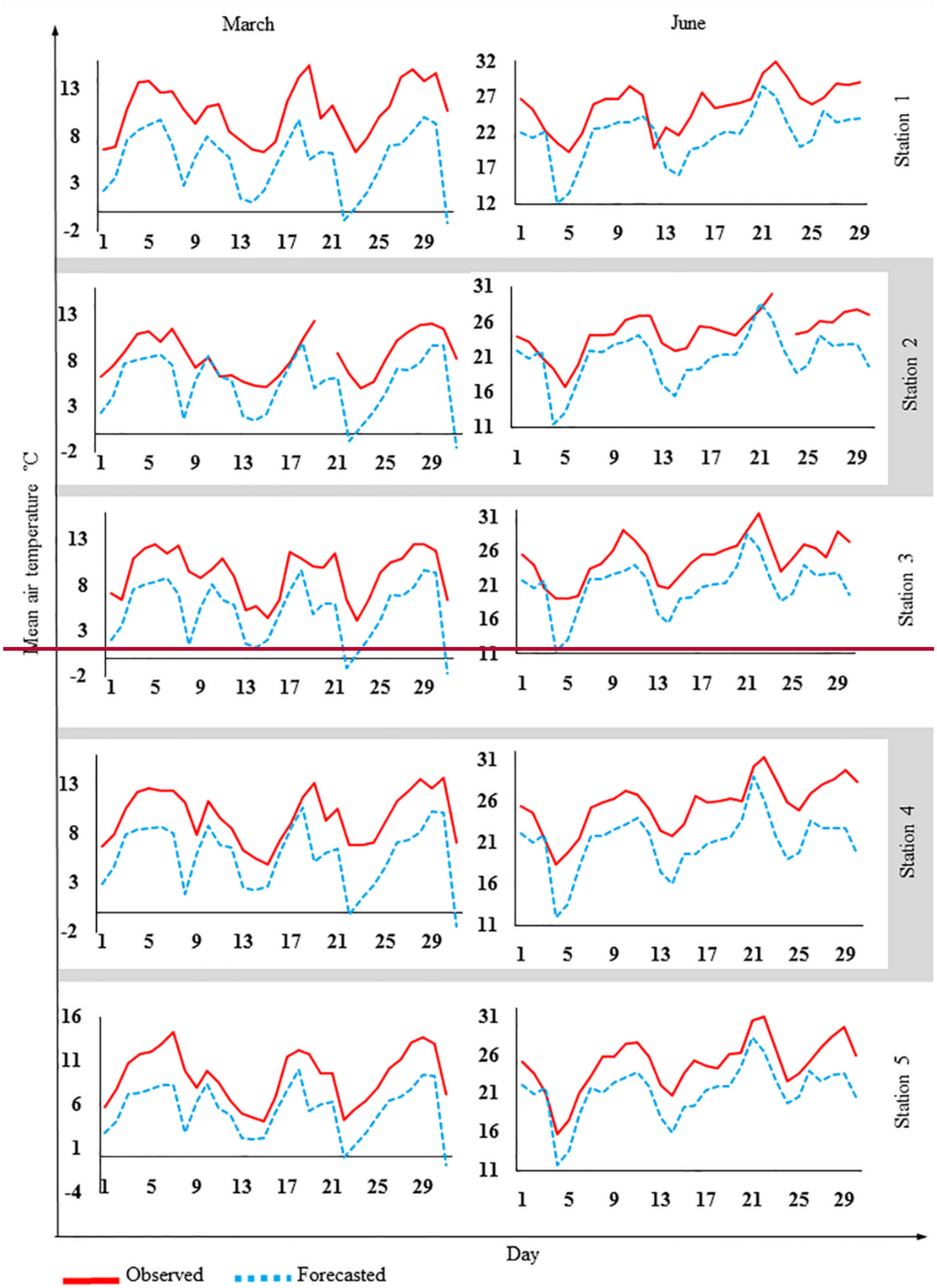
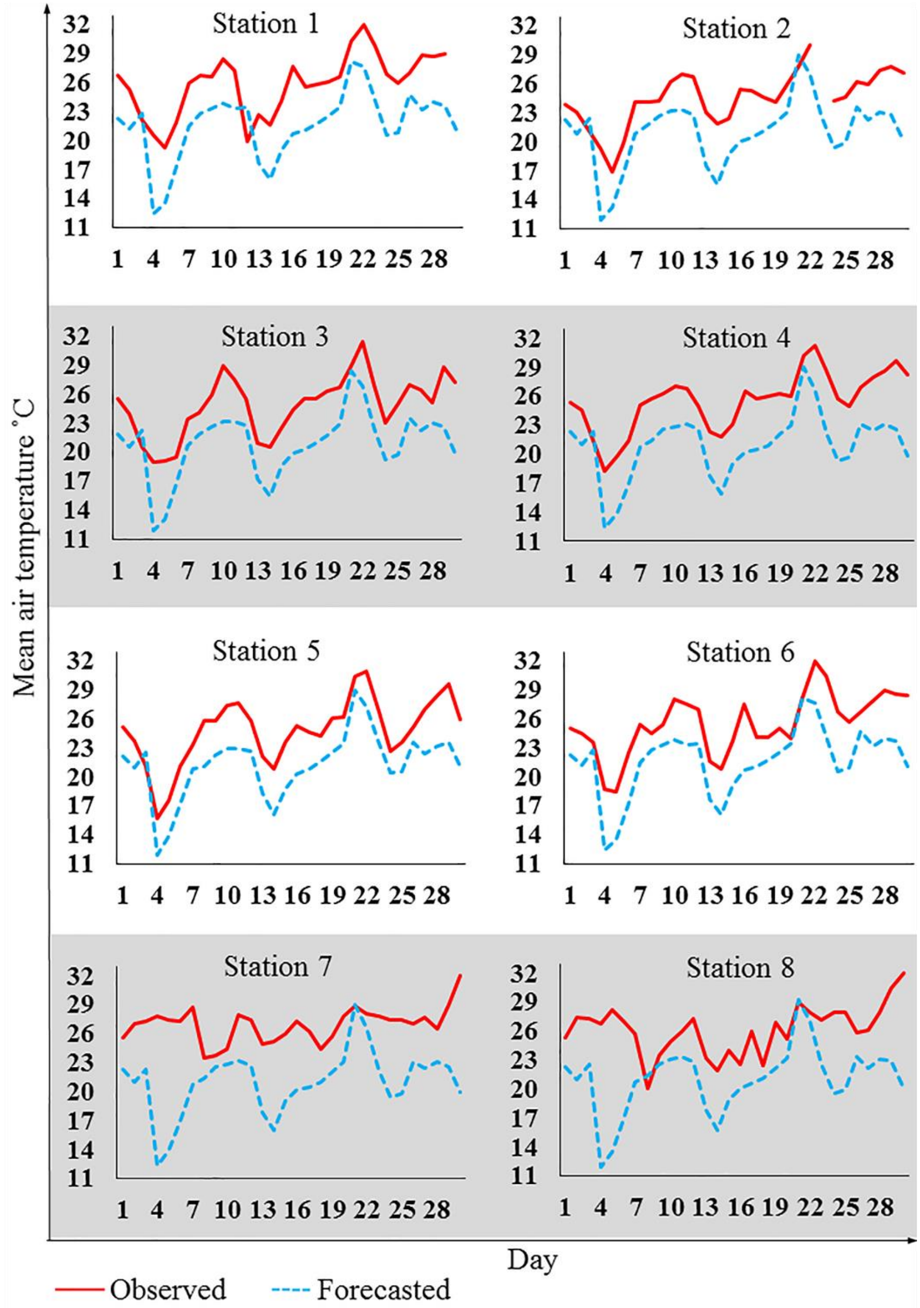
50,2
50,3
50,4
50,5
°E
1
2
3
4
5
Weather Station
ECMWF grid point
Irrigation Network
Study Area

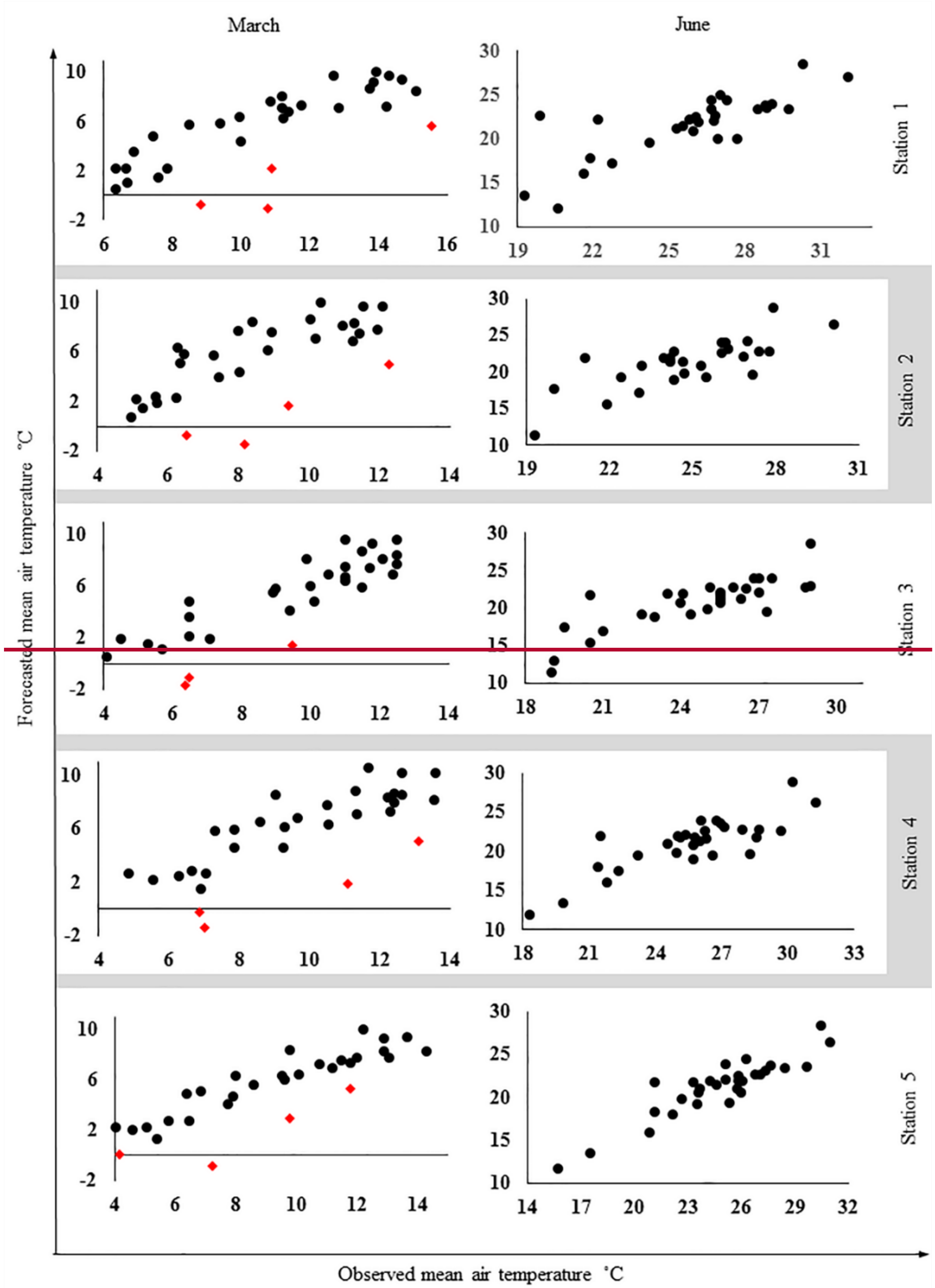


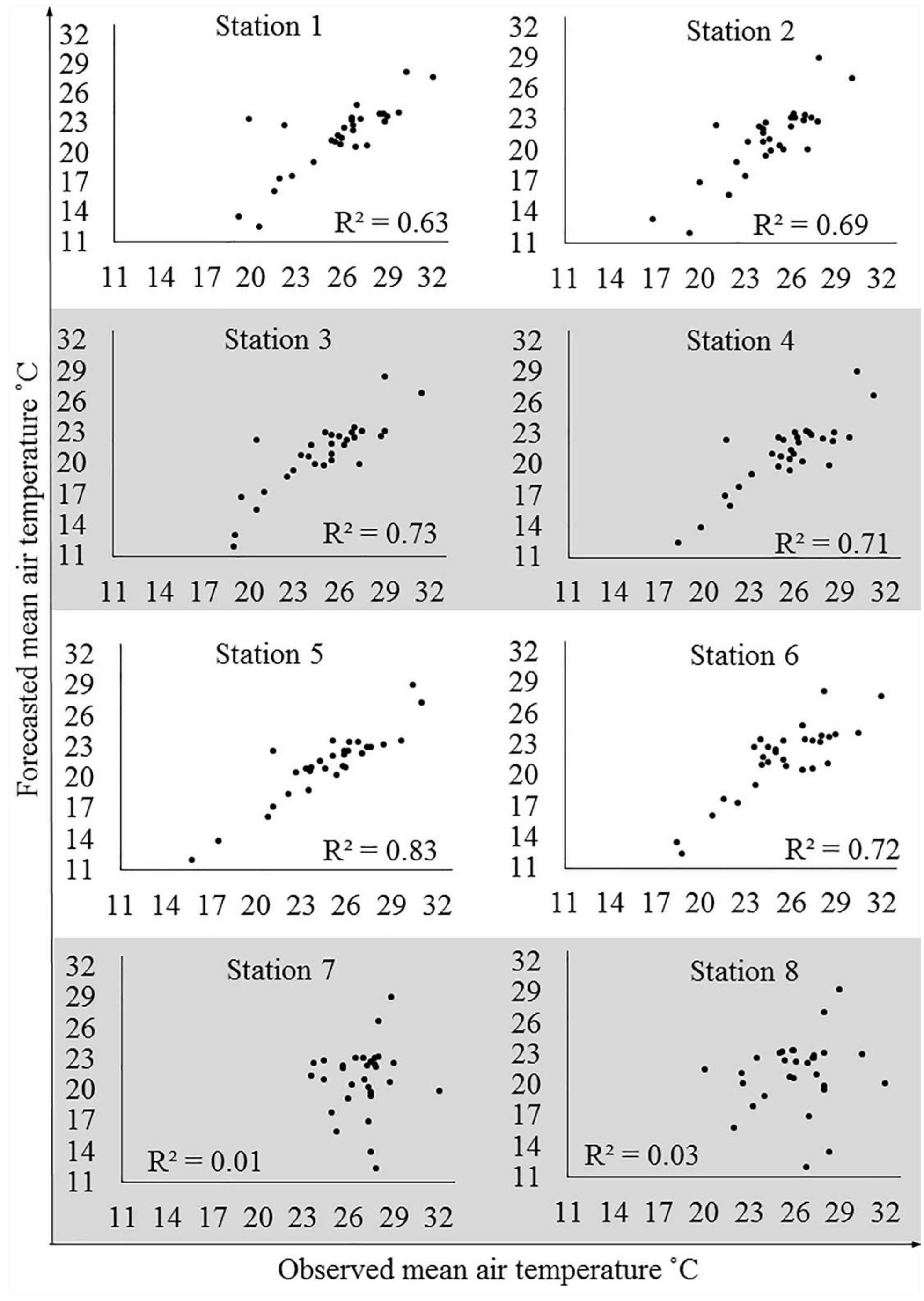
~~Figure 1:~~ Figure 3: Study Area ~~is~~ located in Qazvin, Iran. The~~is~~ area covers the Qazvin irrigation network with a total area of 3307 ~~km2~~ km^2 that is composed of agricultural fields, dominated by the growing of winter and summer crops, urban area, bare soil and natural vegetation. Weather stations are sparse and the minimum and maximum distance between stations are 13 and 78 km, respectively. For experimentation in this study, a sample subset of 3 × 8 grid points of ECMWF dataset is selected at 0.125° lat/lon distances.


March
June
Station 1
Station 2
Station 3
Station 4
Station 5
Mean air temperature °C
Day
Observed
Forecasted

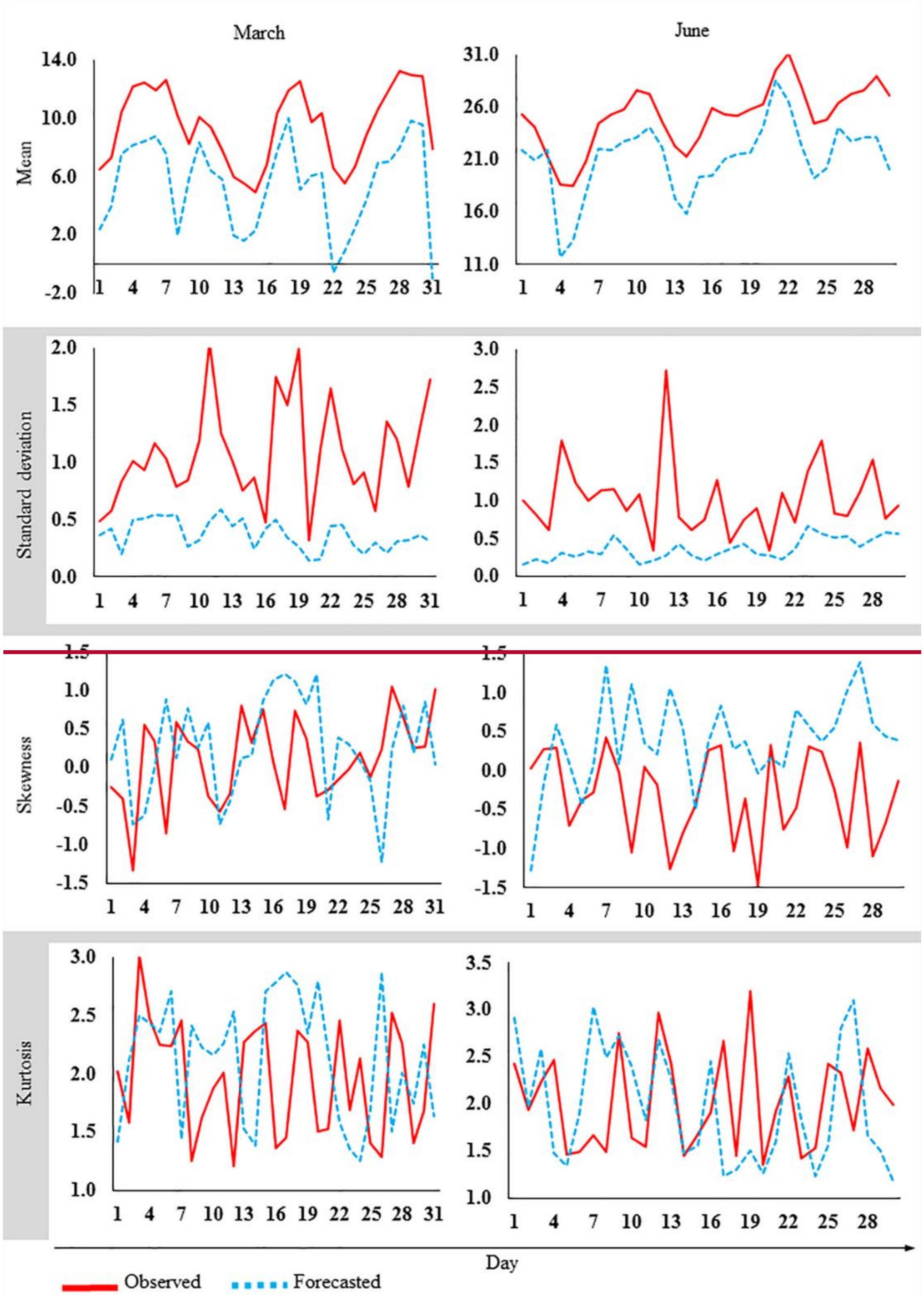


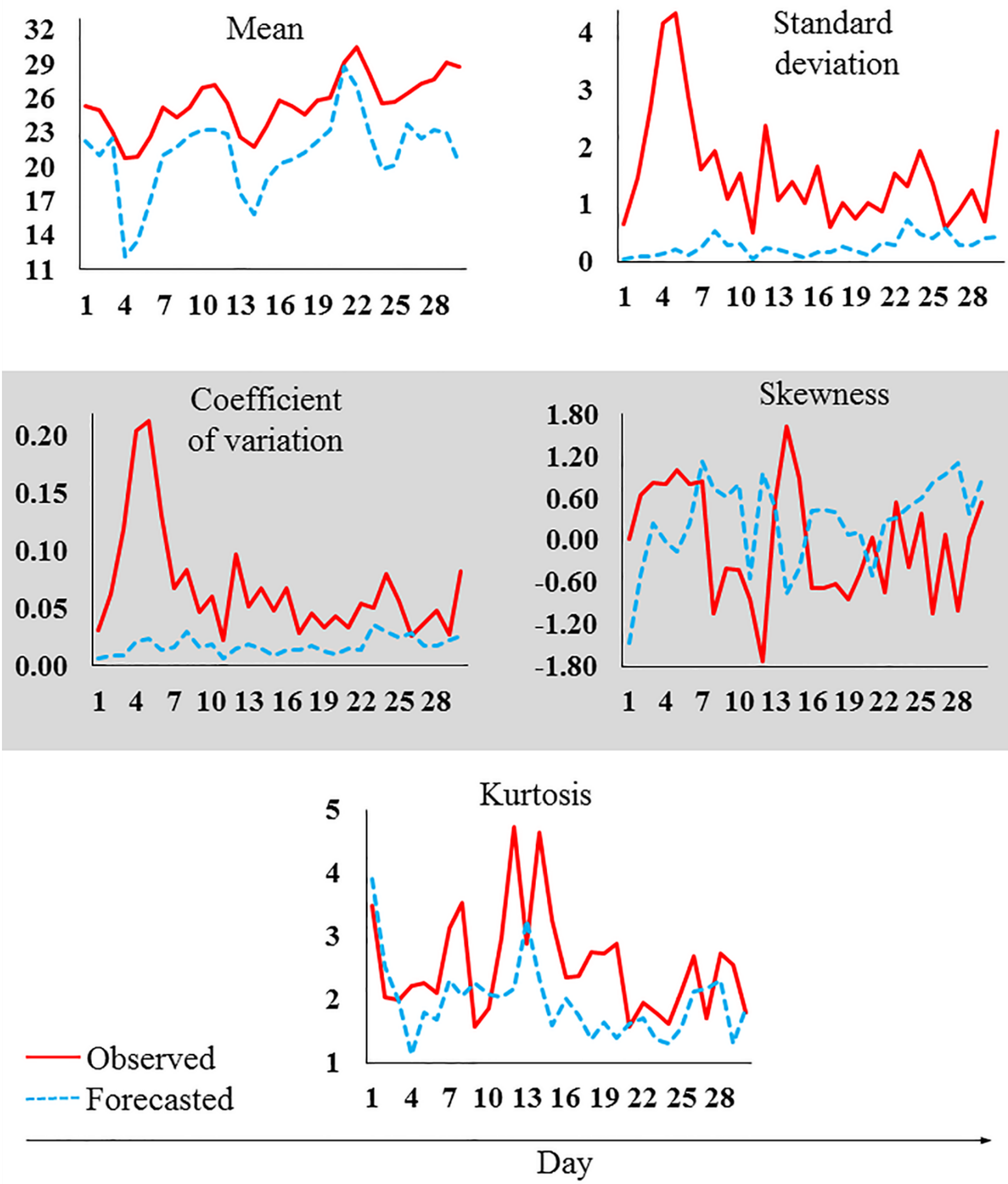
~~Figure 2:~~ Figure 4: Time series of the observed and the forecasted values at each station in June 2014. The observed values are daily air temperature from weather stations and the forecasted values are daily air temperature from ECMWF~~ at same locations~~. ~~This figure shows the underestimation in ECMWF as well as spatial and temporal variability of bias.~~


March
June
Station 1
Station 2
Station 3
Station 4
Station 5
Forecasted mean air temperature °C
Observed mean air temperature °C

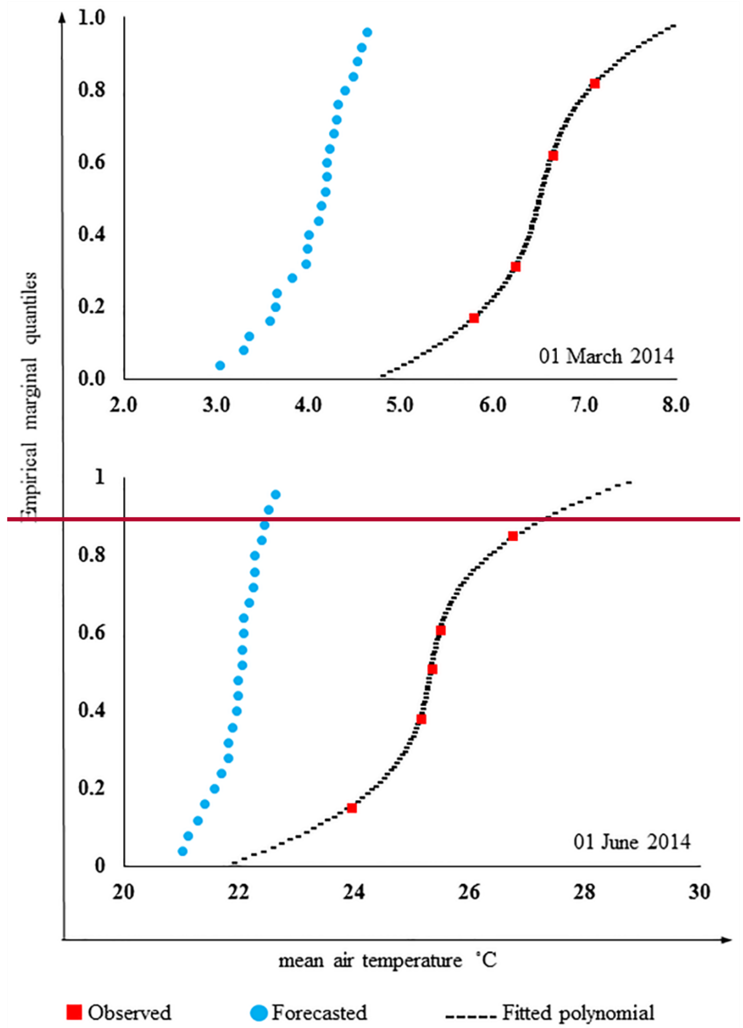


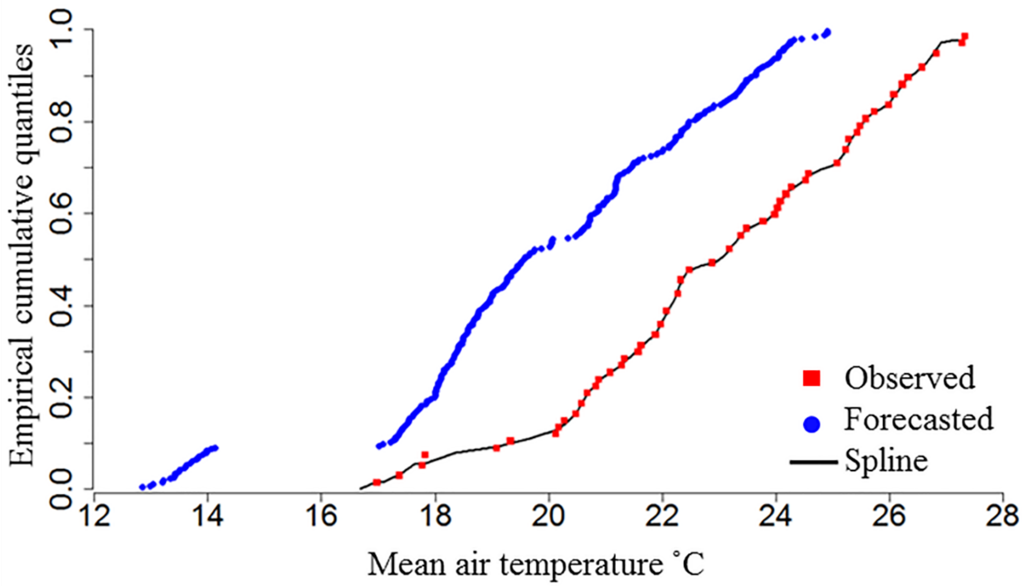
~~Figure 3:~~ Figure 5: Scatterplot ~~of~~ between the observed and the forecasted values at each station during June 2014. The observed values are ~~daily~~ air temperature from weather stations and the forecasted values are ~~daily~~ air temperature from ECMWF. ~~at same locations. Red points in the scatterplot denote the outliers.~~


March
June
Mean
Standard deviation
Skewness
Kurtosis
Day
Observed
Forecasted

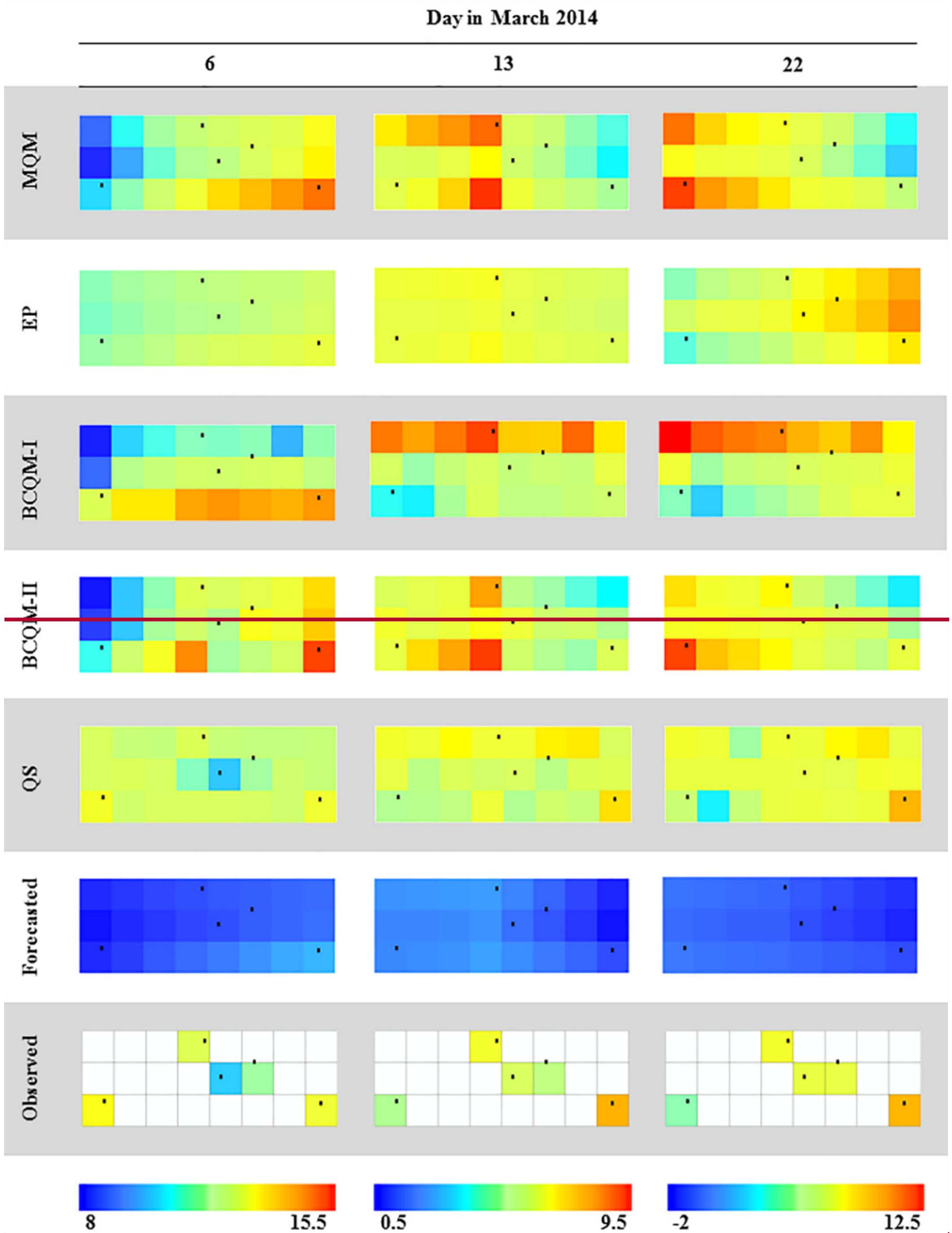


~~Figure 4:~~ Figure 6: ~~Bias between the~~ The sample moments of the observed and forecasted ~~marginals~~ variables at each ~~moment~~ day of time series in June 2014. The observed values are ~~daily~~ air temperature from weather stations and the forecasted values are ~~daily~~ air temperature from ECMWF ~~at same locations~~.

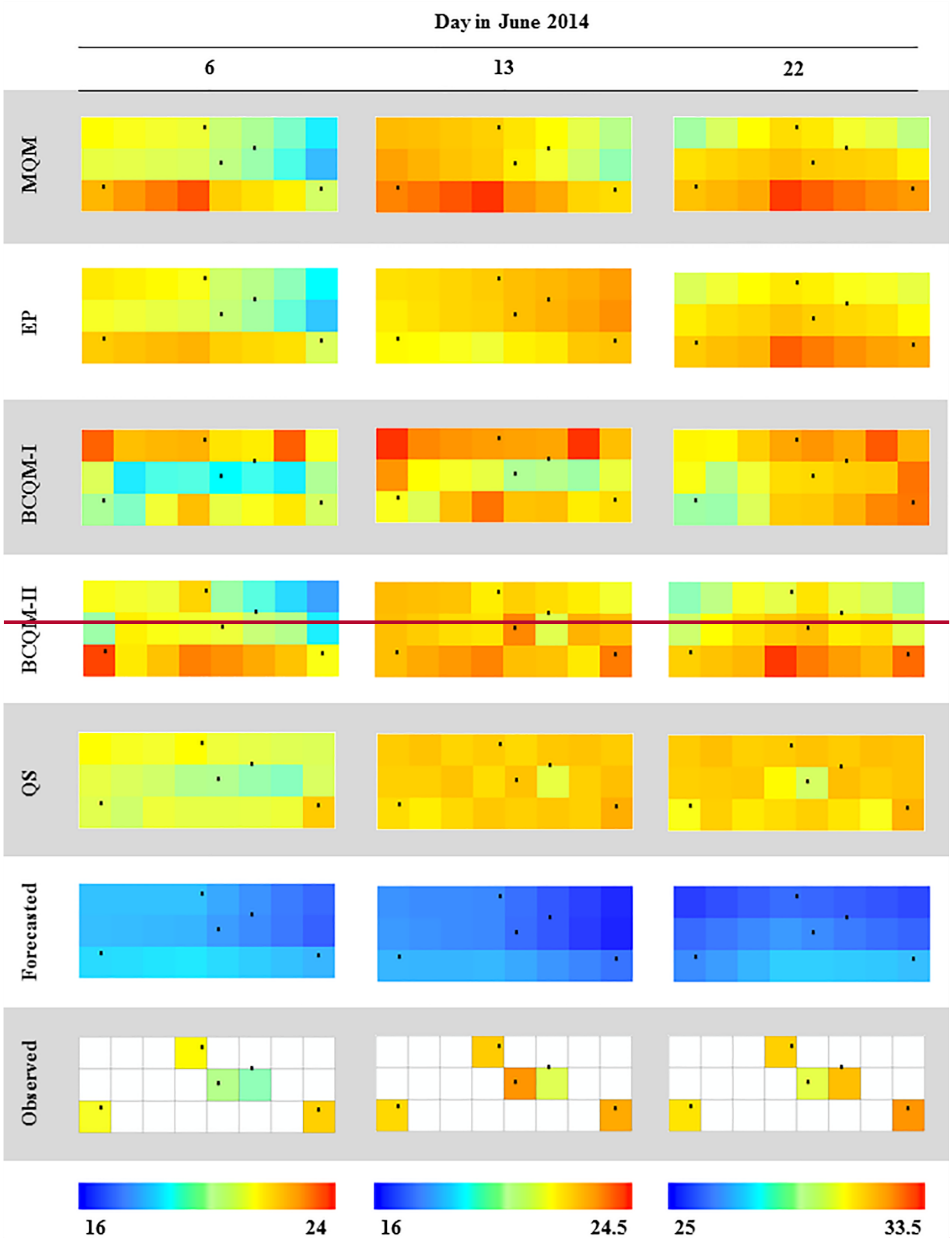

Empirical marginal quantiles
01 March 2014
01 June 2014
mean air temperature ˚C
Observed
Forecasted
Fitted polynomial



~~Figure 5:~~ Figure 7: The empirical marginal quantiles ~~values u_1 and u_2~~ and the fitted ~~polynomial~~ spline for the observed and forecasted air temperature ~~for~~ at ~~first day of March and~~ June 1st 2014.



~~Figure 6: The spatial variability of the observed and the bias-corrected values comparing with the forecasted values over the study area in March 2014. The observed values are daily air temperature from five weather station, the bias-corrected values are the result of the bias correction methods and the forecast values are daily air temperature from ECMWF. For experimentation in this study, a sample subset of 3 × 8 grid points of ECMWF dataset is selected at 0.125° lat/lon distances.~~


Day in June 2014
6
13
22
MQM
EP
BCQM-I
BCQM-II
QS
Forecasted
Observed
16
24
16
24.5
25
33.5

~~Figure 7:~~

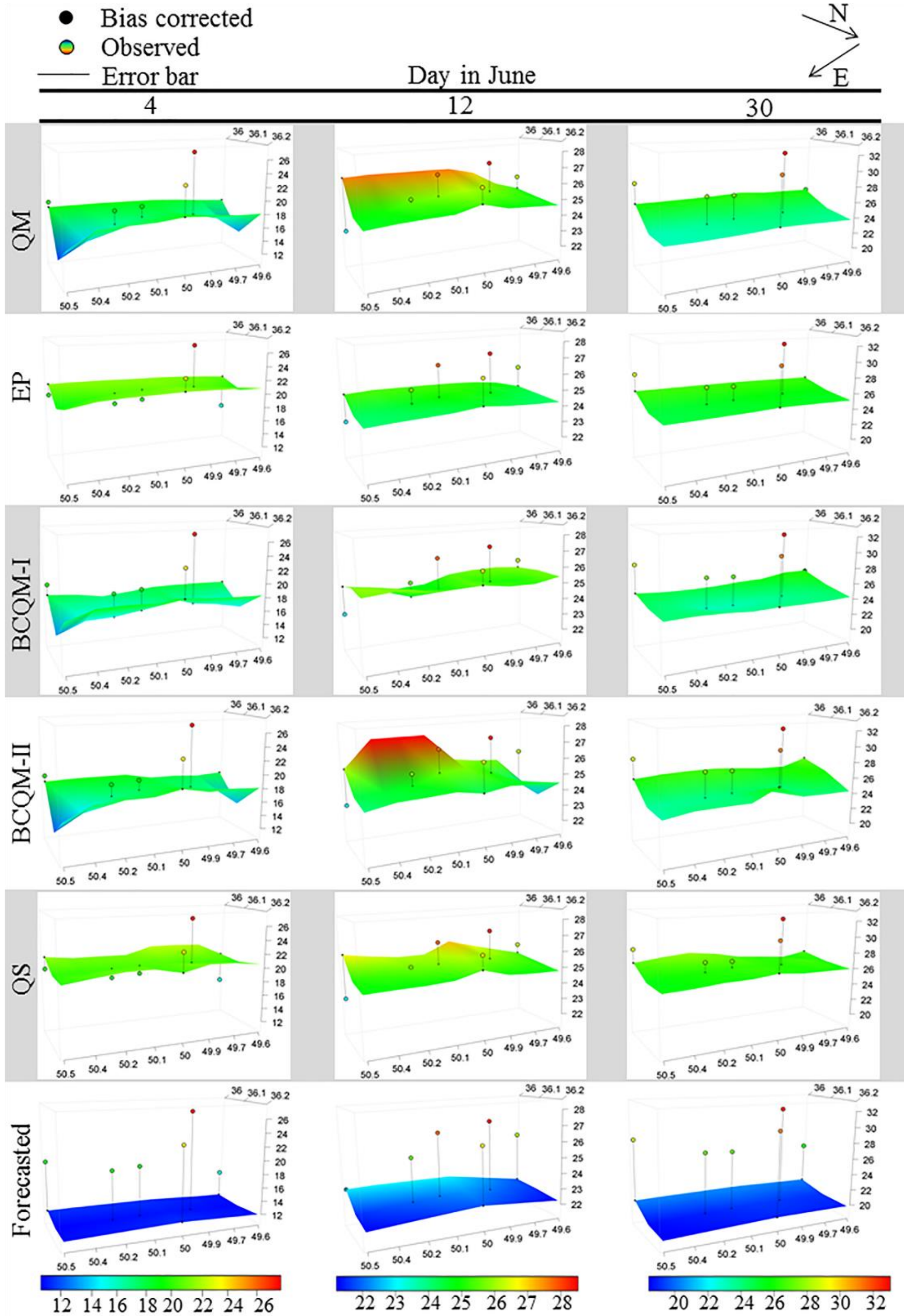


Figure 8: The spatial variability of the observed and the bias-corrected values comparing with the forecasted values over the study area for three selected days in June 2014. The observed values are daily air temperature from ~~five~~ eight weather stations, the bias-corrected values are the result of the bias correction procedures and the forecast values are daily air temperature from ECMWF. For experimentation in this study, a sample subset of 3 × 8 grid points of ECMWF dataset is selected at 0.125º lat/lon distances.