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Nonlinear Processes in Geophysics
Discussions
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Interactive comment on "Detecting Changes in Forced Climate Attractors with Wasserstein Distance" *by* Yoann Robin et al.

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The authors propose the use of the Wasserstein distance in order to discriminate different dynamical systems from their attractors, notably for the case of climate systems. I found the paper really interesting. Moreover, the adoption of such new metric is well motivated and seems really promising for future climatic applications. Thus, I recommend the publication of the manuscript. I have only a general comment and a few specific ones (see below) that could be useful to improve the manuscript.

General comments

Did the authors studied the robustness of the Wasserstein distance to variations of the size of the boxes B_a? I think this is an important point, in particular once that their

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method will be applied to realistic systems. Even though an explicit numerical study is not requested, a discussion of this aspect would be really appreciated.

Response: The value of 0.1 gives 40 to 60 bins on each axis, assuming that the attractor lives in a box of $[-1;3] \times [-3;3] \times [-3;3]$. This means that the volume is divided into $40 \times 60 \times 60$ boxes. This number is the same order of magnitude as the number of gridcells in the NCEP reanalysis around the North Atlantic region, should one be interested in the climate attractor of that region (e.g. Faranda et al (2017),). We also tried values of 0.05, 0.2 and 1.0 for the size of the boxes (so, a factor 2 for the two first, and one scale up for the last) for the protocol of Section 3. For all values, the maximal variation of standard deviation is 0.01, and the detection is not affected. For a size of 0.05 and 0.2 the maximal variation of the median is 0.03. For the size 1.0, the maximal increases of median of box plot of winter (resp. summer) against itself is 0.22 (resp. 0.18), but the difference with the median of winter against summer is at least equal to 0.3.

Modification 1 (Page 5, line 5): We have added the sentence (end of section 3.1): "We chose a bin length of 0.1 for the Lorenz attractor. Therefore $40 \times 60 \times 60$ bins cover the attractor, which remains in a $[-1;3] \times [-3;3] \times [-3;3]$ box. This number of bins is comparable to the number of gridcells that cover the North Atlantic region in the NCEP reanalysis (or most CMIP5 model simulations). This example refers to a few papers dealing with climate attractor properties (e.g. Corti S. et al (1999), Faranda D. (2017)).

Modification 2 (Page 7, line 15):The sentence "This protocol was also applied for bin sizes of 0.05, 0.2 and 1.0. For 0.05 and 0.2, the maximal variation of median (resp. standard deviation) of Wasserstein distances is 0.03 (resp. 0.01), so the distributions are indistinguishable in practice. For a bin size of 1.0, the maximal increase of the median is 0.22, but the difference with the median of winter against summer is at least equal to 0.3." has been added at the end of Sec. 3.3.

Specific comments

(2 Distance between measures - line 14): It could be not easy for any reader how you go from attractors to mass distributions. It would be great to have a short introduction to the definition and use of invariant measures in phase space.

Response: We agree with you.

Modification: We added the sentence "The measure of a sub region of phase space is the probability of a trajectory of the system to go through the region. The invariance is characterized by the conservation of the volume by the dynamics of the system Ruelle (1989)" in Sec. 3.1.

(2 Distance between measures - line 22): Why the authors did not defined (and discuss the differences respect) the Mahalanobis distance?

Response: The Mahalanobis distance was just given as an example of possible distances used in climate sciences. We removed the reference to the Mahalanobis distance, since we do not make any comparison with it.

(2 Distance between measures - line 11 - second paragraph): It would be interesting to know why the authors choose network simplex algorithms to compute the distance. Could be explained why they are better than other classical choices like, for instance, simulated annealing algorithms?

Response: The optimal transport literature classically mentions two kinds of methods: Network Simplex and Entropy Regularization. These two approaches have the advantage to be computationally fast. The Network Simplex is generic but the Entropy Regularization needs a control parameter to be adapted for each system. Thus we have decided to use the Network Simplex for this paper. Annealing algorithms require also to test several control parameters (like acceptance probabilities and temperature) depending on the measures considered. This could be problematic for the computation of thousands of distances between various objects.

Modification (Appendix A): An explanation has been added in Appendix A.

(Algorithm 2): Maybe the authors could give a name to the variable: “total number of boxes B_a” like, for example, K.

Response: The variable “A” (in Require/ Ensure of Algorithm 2) is the total number of boxes.

Modification (Algorithm 2): To clarify, we have replaced A by $40 \times 60 \times 60$ (the total number of boxes of size 0.1 in the domain $[-1;3] \times [-3;3] \times [-3;3]$) and explained that $\mu_a > 0$ for a small number boxes.

(3.2 Protocol line – line 16,17): This sentence is not really clear. It could be expanded a bit.

Response: We agree with you.

Modification (Page 6, line 5) : "We choose to simulate 50 attractors of winter and 50 attractors of summer. We have $50 \times 50 = 2500$ different pairs between summer and winter. For the distances between the 50 attractors of the same season (summers or winters), we only consider $1 \leq (k, k') \leq 50$ pairs with $k < k'$. This means that we have 1225 distances for the winter or the summer. So we have at least 1000 distances per distribution. This is a reasonable sample size for a representative Kolmogorov-Smirnov test."

(3.3 Estimation – line 1): The first sentence is not really clear.

Modification: Normalize Wasserstein distance do not add information in our protocol, so the sentence has been removed.

(3.3 Estimation – line 11 to 15): This point is interesting and could be linked to my general comment: which is the sensitivity of the method respect to N together with the number of boxes B_a? Probably such parameters present an interplay in determining the global robustness of the measure.

Response: See general comment for the question. The global robustness of the empirical measure could be estimated by varying this parameter.

(3.3 Estimation – line 4 – second paragraph): Could the authors specify how they computed the p-values for the KS test? Did they use tables of critical values or simulated numerical p-values?

Response: The KS value is computed with an estimation of the cumulated density function of the two distributions, and the difference. The p-value is given by the asymptotic Kolmogorov distribution. Its cumulative distribution function converges to the supremum of a Brownian bridge B, which can be computed with

$$\mathbb{P}(K \leq x) = 1 - 2\sum_{k=1}^{\infty}(-1)^{k-1}\mathrm{e}^{-2k^2x^2}, \quad K = \sup_{t\in[0,1]} |B(t)|$$

This formula can be found in Marsaglia (2003).

Modification (Page 6, line 3): Reference and explanation have been added.

(4.1 Protocol line – line 31): Could the authors show also here the comparison with the Euclidean distance? Why they did not show such calculation?

Response: We show in Figure 1 below the calculation for Euclidean distance. For $N = 50$ and 100, the maximal difference of the mean (resp. standard deviation) between the period before and after the forcing is 0.002 (resp. 0.002). Furthermore, at least 70% of distances are in the pip of mean $\pm$ standard deviation. So, we can not detect the forcing. For $N = 1000$ and 10000 the maximal modification of mean is 0.004, but the standard deviation is multiplied by a factor 20 (0.0002 becomes 0.005). Even if the forcing is detected, the trajectories of distances are not representative of a linear increasing forcing. This calculation was not added in article because we find the same result of Section 3, and we focus only on the Wasserstein distance.

Modification (Page 10, line 11): This explanation has been added at the end of Sec. 4.

References

Corti S., Molteni F., and Palmer T. N.: Signature of recent climate change in frequencies of natural atmospheric circulation regimes, Nature, 398, 799–802, doi:10.1038/19745, 10.1038/19745, 1999.

Faranda D., Messori G., and Yiou P.: Dynamical proxies of North Atlantic predictability and extremes, Sci. Rep., 7, doi:10.1038/srep41278, http://dx.doi.org/10.1038/srep41278, 2017.

Marsaglia G., Tsang W. W., and Wang J.: Evaluating Kolmogorov's Distribution, Jour. Stats. Soft., Art., 8, 1–4, doi:10.18637/jss.v008.i18, https://www.jstatsoft.org/v008/i18, 2003.

Ruelle, D.: Chaotic Evolution and Strange Attractors, Lezioni Lincee, Cambridge University Press, doi:10.1017/CBO9780511608773, 1989.

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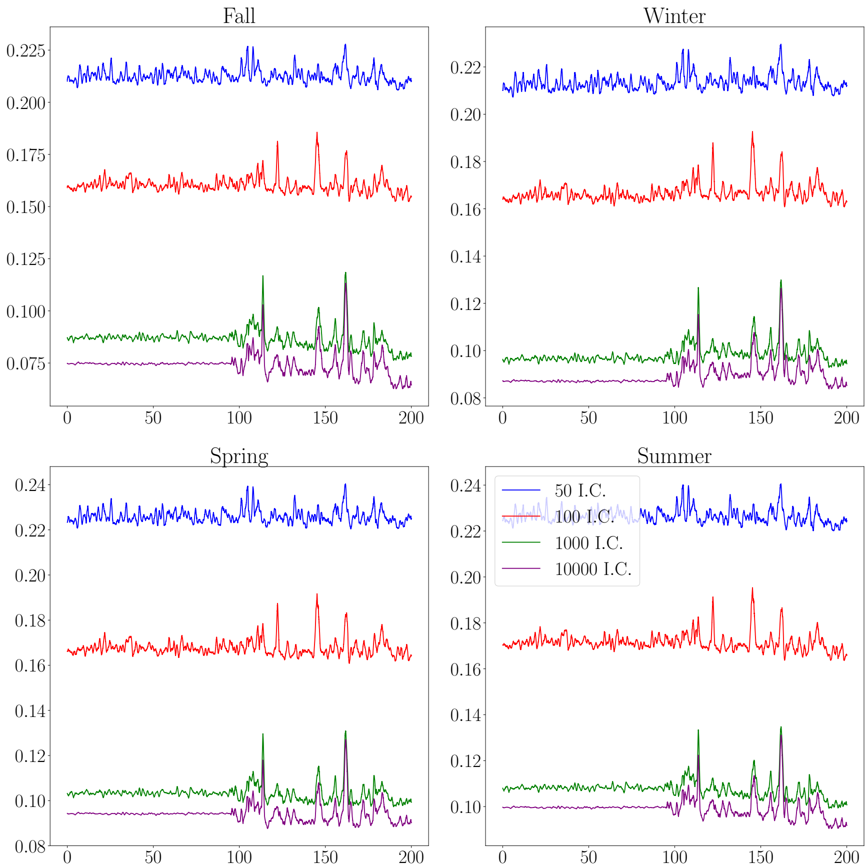

Fall
Winter
Spring
Summer
50 I.C.
100 I.C.
1000 I.C.
10000 I.C.
0.225
0.200
0.175
0.150
0.125
0.100
0.075
0.24
0.22
0.20
0.18
0.16
0.14
0.12
0.10
0.08
0
50
100
150
200

Fig. 1.