

## ***Interactive comment on “Moment-based Metrics for Global Sensitivity Analysis of Hydrological Systems” by Aronne Dell’Oca et al.***

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We would like to thank Prof. Class for the careful and thorough reading of our manuscript and for his insightful comments and constructive suggestions that these have helped to improve the quality of the manuscript. Following is an itemized list of his comments and our responses.

1) This manuscript addresses the improvement of GSA (Global Sensitivity Analysis) by using newly introduced metrics based on the evaluation of the first four statistical moments: expectation value, variance, skewedness, tailedness. These metrics allow to extend the analysis beyond variance-based GSA into the details of the shape of the probability density function (pdf). It allows e.g. to detect when a certain model param-

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eter particularly affects the asymmetry or the tailing of a pdf. The authors motivate and outline their idea in the context of GSA very well and nicely explain the expected benefits of their approach.

We thank Prof. Class for his positive assessment of our work.

2) However, like always, there is no free lunch. The method has the disadvantage that it involves significantly more computational effort, this rendering likely impossible to perform for a complex hydrological scenario of practical relevance. The work-around proposed by Dell’Oca et al is the use of reduced-complexity models or surrogate models; in this case, they use the Polynomial Chaos Expansion technique, which can be viewed as an approximation of a target function by polynomials, where the degree of the polynomials determines (at least up to a certain value) the accuracy of the approximation. Surrogate models like PCE require some smoothness in the targeted model output parameters with respect to the investigated parameters. Otherwise, discontinuities in the model output is difficult to reproduce. Therefore, I suppose, that this might involve the possibility that sophisticated GSA metrics become biased from errors introduced by the surrogate model. Thus, to give added value through several-moment-based GSA, we must first assure that the pdfs from the (surrogate) models are correct, i.e. the model represents the physics in all aspects. The manuscript addresses this issue quite well by some test cases, although I observe that all three test cases are relatively simple and do not provide huge challenges regarding possibly complex hydrological or geological features. The first test case is rather an academic exercise (which has, of course, value in itself), the second is an example where the results are definitely smooth in the parameter space; only the third one involves a heterogeneity, but still not very challenging for a polynomial approximation. I propose, maybe for future work, for example, a scenario where a fluid is injected into a geologic reservoir which has a fault zone in greater distance and where the response in terms of leakage to this fracture is rather a step-function. Would such a scenario be reflected properly by these new metrics? I guess that yes, but would the PCE provide the appropriate

model for it? But clearly, the method proposed here is independent of the choice of the surrogate model.

We thank Prof. Class for pointing out this very relevant issue. As remarked by the Reviewer, in this manuscript we introduce novel metrics for global sensitivity analysis. These metrics can be evaluated making use of the complete model or (if the CPU time associated with the evaluation of the full model is too high) by a surrogate model. We also investigate the error associated with the evaluation of our novel metrics by replacing the complete system model through a gPCE. As proof of concept, we applied the proposed methodology to 3 (relatively simple) test cases. We agree with Prof. Class that any surrogate model (including gPCE) could fail in interpreting all the details of highly non-linear systems. The application of our methodology of analysis to more complex cases is envisioned as a future step. We will revise the opening statements of Section 3 of the manuscripts to highlight these relevant aspects.

3) What if parameters are not uniformly distributed as assumed e.g. in 221-222? E.g. permeability in a fractured rock. How important is this assumption?

Varying the probability density function (pdf) of uncertain model input parameters does not impact the definition of the novel metrics we propose. Otherwise, it may affect the actual results, depending on the test case considered. In the manuscript, we select uniform distributions for model parameters to mimic a scenario where the only information available is the expected range of variability of the parameters. We will highlight this issue in the revised manuscript.

4) A given acceptable level of accuracy for the PCE-based approximation of the new indices requires increasing polynomial order with the order of the statistical moment. I can comprehend this statement in 3.3 (and in the conclusions with the careful wording "might be") from looking at the corresponding figures. But is there also some comprehensible reasoning or proof why this is the case? The authors are mathematicians and might be able to understand and explain this issue.

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A rigorous and general proof of the findings encapsulated in Figures 2 and 5 and evidenced by Prof. Class requires, for example, deriving (a) the analytical format of the probability density function (pdf) of a target model output on the basis of its gPCE based approximation at a given order  $w$ , and (b) the corresponding pdf associated with the full system model. We have shown a way to derive the former in a closed-form in a previous work (Riva et al., 2015, already referenced in the manuscript), at least for  $w = 2$ . With reference to the latter, one would need to formulate and solve (either analytically or numerically) appropriate (and exact) system equations satisfied by the pdf of interest or by its moments. This step is still an active area of research within which our group has considerable expertise. Approaches which are available in the literature always require approximations and specific assumptions, which are typically problem dependent. In this framework, a generalization of our findings is out of scope of the current study. Here, we rely on the direct solution of the full system model and the associated gPCE approximation to derive quantitative results for the examples considered. We will clarify these concepts in the revised manuscript.

## References

Riva, M., Guadagnini, A. and Dell'Oca, A.: Probabilistic assessment of seawater intrusion under multiple sources of uncertainty, *Adv. Water Resour.*, 75, 93-104, <http://dx.doi.org/10.1016/j.advwatres.2014.11.002>, 2015.

5) I understood that the analysis helps better identifying the relative importance of individual parameters. There is enormous practical relevance for this, e.g. it would help to prioritize exploration efforts for a particular parameter when it becomes clear from the sensitivity analysis that knowledge about this parameter reduces uncertainty drastically. The authors might want to emphasize such examples a bit more prominently if they like.

We thank Prof. Class for this very interesting comment with which we fully agree. We will add an appropriate paragraph in the revised Introduction.

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6) In the first conclusion, line 475, I would write: "The CALCULATED sensitivity of a model output ..."

We will modify the text accordingly.

7) Very few language issues: 201-202: ... are affected by model uncertain parameters collected in  $x$ . Here is something wrong with the grammar, or I don't understand the meaning.

The sentence will be rephrased as "are affected by uncertain model parameters collected in the parameter vector  $x$ ".

8) 326: maybe "characterize" is here not the right wording? What about "control"?

The key point is to emphasize that the parameters we consider are affected by uncertainty. We will replace the wording 'characterize experimentally' by 'assess experimentally'.

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